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Endogenous Competence and a Limit to the Condorcet Jury Theorem

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Abstract

The seminal contribution, known as the Condorcet Jury Theorem, observes that under a specific set of conditions an increase in the size of a group tasked with making a decision leads to an improvement in the group's ability to make a good decision. An assumption under-appreciated is that the competency of the members of the group is assumed to be exogenous. In numerous applications, members of the group make investments to improve the accuracy of their decision making (e.g. pre-meeting efforts). We consider the collective action problem that arises. We show that if competence is endogenous, then increases in the size of the group encourages free riding. This trades off with the value of information aggregation. Thus, the value of increased group size is muted. Extensions illustrate that if committee members are allowed to exit/not participate, then the equilibrium committee size is reduced. Additionally, (non-decisive) supermajority voting rules encourage the investments and, consequently, individual competence.

Keywords: committee decision making, Condorcet Jury Theorem, endogenous competence, group size, majority voting, supermajority voting

JEL codes: D71, D02, H41

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1 Introduction

The Condorcet Jury Theorem (Condorcet, 1785; Black, 1958) states that a group is better at making decisions than individuals. Specifically, if a committee of imperfectly-informed individuals uses simple majority voting to identify which of two options is the better one, then the voting mechanism aggregates information well so that the accuracy of the group outcome exceeds that of any individual within the group. Furthermore, as the size of the committee grows, the accuracy of the group outcome expands (for reviews of the literature see Mueller (2003) and McCannon (2015)). In the limit, the group identifies the correct option with probability one.

Given this result, expanding the number of individuals on a committee monotonically improves the accuracy of the outcome decided upon. Why, then, don't we see all decisions made by the universal set of individuals? Condorcet himself was a proponent of universal suffrage, including enfranchisement of women and the abolition of slavery (Gregory, 2010). Delegation is common. Whether within a firm, an organization's committee, or in legislative and legal settings, it is typical that a subset of the population is chosen to make decisions. To determine optimal group size, researchers have relied on costs to group formation to balance the benefits of group size (McCannon, 2011) or introduced adjustments to the theoretical environment, such as interdependent information (Boland, 1989; Berg, 1993; 1996) or pre-voting deliberation (Helland and Raviv, 2008), to create a trade-off between group size and accuracy.

We document a limitation to the seminal result. We show that if one first considers an investment decision in competence by individuals, then the endogeneity of intelligence sufffers from a free-riding problem. Competence is a public good. Adding more members to the group discourages investments in it. We show that as the group size grows, these investments shrink.[1] This mitigates

[1] We are not the first to recognize the potential problem with endogenizing competence. Ben-Yashar and Nitzan (2001) raise the issue. They limit their analysis to a numerical example between a committee of three and a single decision maker and document a reduction in aggregate competence when a social planner sets the effort investment. Hao and Suen (2009), in a review of committee decision-making models, also points out that the accuracy of a group's

the information aggregation benefit.

Numerous applications support this proposed extension. For one, suppose an organization forms a hiring committee to make the decision on whom to hire (e.g. a new faculty member). Each member of the committee must devote time and effort investigating the qualifications of the candidate. This means, for example, contacting references and evaluating past performance (e.g. published research). These activities are time consuming. If a committee member has a vote amongst a large group, then the probability the individual's vote will swing the outcome is quite small. Thus, the individual does not have a strong incentive to invest the effort to make sure his vote is informed.[2] Alternatively, if the individual is the sole decision maker (i.e., dictator), then he is incentivized to spend significantly more effort to ensure a good decision is made.

The information aggregation of increased group size trades off with less competence of the members. We show that for any fixed group size, aggregate accuracy is reduced due to free riding. Thus, the marginal benefit to increased committee size is less. Rather, endogenous competence generates a limitation on the Condorcet Jury Theorem. Second, we are able to recover the asymptotic version of the Condorcet Jury Theorem in that, even with the collective action problem prevalent, the probability the group reaches the correct decision goes to one as the group size goes to infinity.

Since its inital exploration (Kazman, 1973; Grofman 1975; 1978), a substantial literature has been established exploring the impact of relaxations to the environment. Noteable examples include heterogenous accuracy (Miller, 1986; Berend and Sapir, 2005; Congleton, 2007), interdependent information (Boland, 1989; Berg, 1993; 1996), polychotomous choice (Hummel, 2010), supermajority voting rules (Fey, 2003), and strategic voting (Feddersen and Pesendorfer, 1996; Ben-Yashar, 2006). The seminal result has been applied to decision-making

outcome can be worsened if the informational signals are acquired with a cost. They do not explore the interaction between this and group size. Additionally, the potential problem is hypothesized in McCannon (2015).

[2] Thus, the literature on rationally-ignorant voters (Caplan, 2007) is a useful reference. We differ, though, in that we formally explore how the quality of voter decision making adjusts to changes in the size of the population.

problems in health (Koch and Ridgley, 2000; Gabel and Shipan, 2004) and arbitration (Marselli, McCannon, and Vannini, 2015), to name two examples. The information aggregation has been used to justify free speech (Ladha, 2012), jury decision making (Klevorick and Rothschild, 1979; Coughlan, 2000; Hummel, 2012), and an extension of the franchise (Ben-Yashar and Zahavi, 2010). We contribute to this literature by highlighting the public goods nature inherent in group decision making.

An early, noteworthy discussion of this phenomenon is introduced by Tullock (1971). Considering judges, as an opening example, he points out that through their decisions they provide a public good. The net gain from a well-reasoned decision is enjoyed by society, while the judge himself receives relatively little of the benefit. Tullock points out that an "important aspect of the problem concerns the energy and thought the judge puts into making the decision" and he expects that judges will underinvest in these private expenditures. Hence, our contribution can be thought of as further exploring this public good nature of decision making.[3] We expand upon his reasoning by specifically studying how the quality of the choices made responds to the size of the group making the decision.

Finally, we consider two extensions to the environment to explore the consequences of endogenous competence. If members have an initial-stage choice to exit the committee, we argue that under reasonable circumstances the equilibrium number of individuals who stay on the committee is less. This is not driven by costs to committee participation, but rather the recognition that by staying on a committee others are disincentived from investing in the quality of their decision making. Thus, exit encourages those remaining in the group to provide a positive spillover that the individual benefits from. As a consequence, the

[3] Tullock's main argument is that the belief that public goods problems in private markets require government intervention is incomplete. The government bureaucrats also suffer from the free-riding problem and, therefore, it is unclear whether it is an effective way to deal with public goods. As an example, he compares a designer of bumpers on Chevrolets to a politician. Both gain only a tiny fraction of the benefits they create, while incurring private costs. The primary difference between the two is General Motors has the incentive to monitor and punish ineffective bumpers, but voters too suffer from free riding when holidng politicians accountable.

public good nature of competence explains delegation to smaller committees.

Second, we consider the extension to supermajority voting rules and identify a previously undocumented result. The probability the group is tied is more likely for a supermajority threshold than simple majority. This acts to incentivize the competence investments and for a fixed group size improves individual decision making. While increases in group size still facilitate free riding, individual competence and at each level of group size, is higher.

Section 2 presents the theoretical model, while Section 3 derives the equilibria. To illustrate the results, a numerical example is provided in Section 4. Section 5 extends the environment to allow for exit and to incorporate supermajority rules. Section 6 concludes.

2 Model

A group is made up of N members, where N is odd (to rule out the possibility of a tie). Individuals in the group are indexed i with $i = 1, 2, ..., N$. The group uses simple majority voting (without abstention) to implement one of two options labeled A and B. Let σ denote the state; $\sigma \in \{A, B\}$. For simplicity, assume the probability of each state occurring is equally likely. Denote z as the decision made by the group, or rather, the option that is selected using the simple majority vote.

Each member of the group has the same common preferences. The payoff to individual i, $u_i(z)$, is greater if the selected outcome matches the state, $u_i(z = \sigma) > u_i(z \neq \sigma)$. Thus, each individual is interested in selecting the "correct" outcome. The benefit of making the correct decision is $u_i(z = \sigma) = b > 0$. If the wrong outcome is selected ($z \neq \sigma$), the payoff is normalized to zero.

When voting, individual i accurately assesses the state of the world with probability p_i. We assume competence is independent of the state of the world and is independent of competence of other members of the group.[4]

[4] These are, of course, strong assumptions, but standard in the literature. For an overview discussion of the role of the interdependence in competence see McCannon (2015). One would expect that free riding can be exacerbated if there is also positive serial correlation. This extension, though, is not considered here.

Prior to the voting stage, assume an initial investment in competence is selected. For example, members of a committee can invest time and effort into investigating and researching the options. Thus, individuals play a two-stage game. Let e_i denote the initial-stage effort investment made by individual i. Assume $e_i \geq 0$ and the choices are made simultaneously by the N group members. Let E denote the profile of investments made by the N players, $E = (e_1, e_2, ..., e_N)$.

Therefore, the probability of making the correct decision in the voting stage is a function of the investment made, $p_i \equiv p(e_i)$. We assume each agent is endowed with the same competence production function[5], but can endogenously choose to differ in their decision-making ability. To keep the model simple and tractable, we assume the function $p(e)$ is continuously differentiable, strictly increasing, and strictly concave. Thus, an increase in e increases an individual's competence, but with diminishing returns. Furthermore, as is common in the literature, we assume there is a lower bound to the competence, which is strictly greater than one-half. Rather $p(0) = \underline{p} > 1/2$.

Additionally, the investment is costly. Let $c(e_i)$ denote the cost of the investment by player i. Assume the cost function is continuously differentiable, strictly increasing, and strictly convex. Again, we assume individuals have identical cost functions. Finally, assume $\frac{dc}{de} = 0$ when $e = 0$, so that small levels of effort come at very little cost, and $\frac{dc}{de} \to \infty$ as $e \to \infty$ so that marginal cost escalates for very high levels of effort. These assumptions are made to simplify the analysis in that they guarantee an interior solution exists.

Thus, the timing of the game is straightforward. First, the N ex ante identical members of the group simultaneously make investments in their competence. This results in a profile E where agent i has a competence of $p(e_i)$. In the second stage each individual casts a vote for one of the two options. We assume

[5]Individuals in practice differ substantially in their ability to acquire information and make good decisions. The assumption of identical competence production functions is done for convenience. Similarly, individuals may differ in the benefit they receive from the correct decision (or, relatedly, the disutility from an error), which would incentivize heterogeneous investments. The main argument (that there exists a free-riding problem in group decision making that is exacerbated in larger groups) can be expected to hold in extended environments.

the individuals vote sincerely in that if they receive a signal option A, for example, is correct, then they vote for it.[6] The option voted for by at least $\frac{N+1}{2}$ of the individuals is selected and, if this majority is correct, each player earns the benefit b.

We assume the agents are not able to form enforceable contracts on the effort investment made in the initial stage. One can think of it as unobservable effort, which frustrates contracting, or unverifiable activites where a contract formed cannot be enforced. Furthermore, the unknown competence excludes weighted voting rules or delegation to an expert.

3 Equilibrium

Consider the payoff function for individual i. The expected payoff is

$$\pi_i(e_i) = bP(E) - c(e_i) \tag{1}$$

where $P(E)$ is the probability the group selects the correct outcome. If, for illustration purposes, each person selects the same investment generating a competence of p, then

$$P(E) = \sum_{x=\frac{N+1}{2}}^{N} \frac{N!}{x!(N-x)!} p^x (1-p)^{N-x}. \tag{2}$$

Equation (2), though, is illustrative in that it depicts the probability calculation when homogeneous competence occurs. This need not necessarily be selected by the actors. Given this payoff structure, players must weigh the benefits of selecting the correct outcome with the costs of putting in more effort to increase the likelihood of getting that outcome.

[6] We make this assumption to minimize the difference between the model presented here and the standard framework employed in the literature. With strategic voting with abstention, as in Feddersen and Pesendorfer (1998), one would expect the decision to abstain by others to affect the effort investment. See Austen-Smith and Banks (1996) and Ben-Yashar (2006) for discussions of the impact of strategic voting.

3.1 Symmetric Nash Equilibrium

We restrict attention to symmetric Nash equilibria of the model where each individual selects the same investment and, hence, has identical competence. This is a reasonable refinement given that the competence production function and the cost function are identical for all players.[7] We first establish that such an equilibria exists in the model.

To do so, we need to identify the best response for an individual. While the investment made incurs cost, the vote cast has a marginal impact on the outcome only when the other $N-1$ voters have voted in such as way as a tie occurs. Thus, define T_i^N as the probability the $N-1$ players, other than i, are tied. Hence, in a symmetric outcome, for example,

$$T_i^N = \frac{(N-1)!}{\left(\frac{N-1}{2}\right)!\left(\frac{N-1}{2}\right)!} p^{\frac{N-1}{2}} (1-p)^{\frac{N-1}{2}}. \tag{3}$$

The derivation of T_i^N in (3) holds when the individuals have selected the same investment and, hence, have identical competence. As stated, in general this need not be the case. Again, the presentation of T_i^N is for illustrative purposes.

Using T_i^N, the expected payoff to individual i can be rewritten as

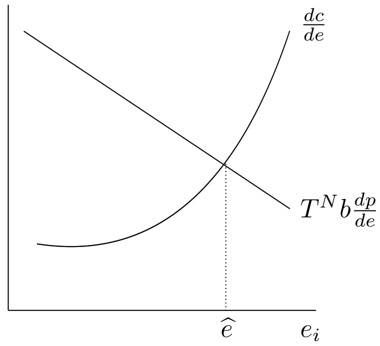
$$\pi(e_i) = T_i^N p(e_i) b + \left(1 - T_i^N\right) Z - c(e_i) \tag{4}$$

where Z is the expected return controlled by the other individuals (when i's vote cannot impact the outcome). The best response for i, then, is the one that maximizes $\pi(e_i)$, or rather, since T_i^N and Z are not functions of e_i, $\frac{d\pi}{de} = T_i^N b \frac{dp}{de} - \frac{dc}{de} = 0$. Therefore, for an interior solution the best response is

$$T_i^N b \frac{dp}{de} = \frac{dc}{de}. \tag{5}$$

[7] One would expect there to be numerous asymmetric Nash equilibria in this environment as well. For example, a subset N' may choose a low level of effort, while the rest choose a high level. Those in N' will see a reduced chance of influencing the outcome (since N/N' are likely to reach the correct decision) and, therefore, the low investment is justified. Those in N/N' will anticipate a higher probability of being able to swing the outcome and, hence, are willing to invest more in competence. The main result, though, can still be expected to hold as an increase in the group size reduces the probability of being able to break a tie for those in both groups.

Figure 1: Symmetric Nash Equilibrium



Consequently, define the investment $\widehat{e}$ and profile $\widehat{E}$ as the symmetric outcomes that satisfied (5) each each individual i. Figure 1 graphically illustrates the first-order condition.

Proposition 1 establishes the existence and uniqueness of such an outcome.

Proposition 1. *A unique symmetric equilibrium exists.*

Proof. Consider all symmetric profiles $E^s = (\epsilon, ..., \epsilon)$. Define $S_i^N(\epsilon)$ as the probability $N-1$ players each selecting ϵ result in a tie. Thus, the function S_i^N is T^N restricted to the domain of symmetric investment profiles. It follows that since $p(e)$ is a continuous, strictly increasing, strictly concave function, then $S_i^N(\epsilon)$ is also a continuous, strictly decreasing (since $p(1-p)$ is strictly decreasing in e), strictly concave function. As a consequence, $S_i^N b \frac{dp}{de}$ is continuous and strictly decreasing. Also, it is assumed that $c(e)$ is continuous, strictly increasing, and strictly convex. Additionally, it is assumed that $\frac{dc}{de} = 0$ at $e = 0$. Hence, the Intermediate Value Theorem guarantees a crossing point (i.e., existence) and, since the former is strictly decreasing while the latter is strictly increasing, a single crossing point exists (i.e., uniqueness). □

Now that it is established that a unique symmetric equilibrium exists, the properties of it can be explored. First, Lemma 1 establishes that ties are less likely when the group size is larger.

Lemma 1. *For a fixed (symmetric) E, the probability of a tie is decreasing in N. Furthermore, as $N \to \infty$, the probability of a ties goes to zero.*

Proof. Fix $E = (\epsilon, ..., \epsilon)$. Since it is assumed that $p(\epsilon) > 0$ for all ϵ, $S_i^N > 0$. Also, note that $2S_i^N p(\epsilon)(1 - p(\epsilon)) = S_i^{N+2}$. Since $p(\epsilon) \in \left(\frac{1}{2}, 1\right)$, it follows that $p(\epsilon)(1 - p(\epsilon)) \in \left(0, \frac{1}{4}\right)$. Hence, $S_i^N(\epsilon) > S_i^{N+2}(\epsilon)\ \forall \epsilon$. Furthermore, it follows immediately that as N increases, then, for any positive value λ, there exists a finite value n such that $S_i^n < \lambda$. Hence, as $N \to \infty$, $S_i^N \to 0$. □

Given this, the main result can be established.

Proposition 2. *The symmetric equilibrium investment is strictly decreasing in N.*

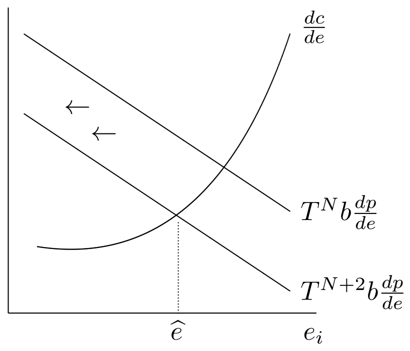
Proof. The equilibrium condition requires $T_i^N b \frac{dp}{de} = \frac{dc}{de}$. In a symmetric Nash profile $T_i^N = S_i^N$. From Lemma 1, an increase in N strictly decreases the left-hand-side of the equilibrium condition. Since $c(e)$ is strictly convex, this implies that e_i must decrease in equilibrium. Similarly, since $p(e)$ is strictly concave, this also implies that e_i must decrease in equilibrium. Thus, the (unique) symmetric equilibrium is strictly decreasing in N. □

Thus, if one allows for competence to be determined endogenously by the committee members, then free riding is a concern. Investments in the ability to make a good decision creates a non-rival and non-excludable spillover to the others in the group. A larger group size magnifies the free-riding problem. Thus, increases in group size lead to lower investments and less competence. Figure 2 graphically depicts the result.

Therefore, the result illustrates the free-riding nature to decision making in a group when competence is endogenous. This acts as a limitation to the Condorcet Jury Theorem. The natural question, then, is whether the seminal result still holds. Proposition 3 provides an answer.

Proposition 3. *As $N \to \infty$, the probability the correct outcomes is selected goes to one; $P(\widehat{E}_N) \to 1$.*

Figure 2: Increase in Group Size



Proof. Consider the first-order condition presented in (5). From Lemma 1 as $N \to \infty$, $S_i^N \to 0$. Consequently, since $\frac{dc}{de}$ is strictly increasing and $\frac{dp}{de}$ is strictly decreasing, $\widehat{e} \to 0$ as $N \to \infty$. As $\widehat{e} \to 0$, $p(\widehat{e}) \to \underline{p}$. Notice that $P(\widehat{E}_N)$ is bounded from below by $\underline{P}$ where $\underline{P}$ is defined by (2) with $p = \underline{p}$. Also, obviously, it is bounded from above by the constant function of $\overline{P} = 1$. Since $\underline{P} \to 1$ as $N \to \infty$, then as a consequence of the Sandwich Theorem[8], $P(\widehat{E}_N) \to 1$ as $N \to \infty$. □

The asymptotic version of the Condorcet Jury Theorem still holds. An important assumption of the model is that even with a zero investment, competence is bounded above $\frac{1}{2}$. Thus, the violation of the asymptotic Condorcet Jury Theorem when adding fair coins, pointed out by Paroush (1988), is avoided.

3.2 Numerical Example

To illustrate the impact of free riding on group decision making, we evaluate a numerical example. For it, we assume

$$p(e) = \alpha - \frac{\beta}{e+1}. \tag{6}$$

[8]The alternative name for this theorem is the "Two-Policemen-and-a-Drunk Theorem" in that if two policemen are taking a drunk prisoner to his cell, then regardless of the amount of wobbling that goes on, if the guards keep the prisoner between them and they make it to the cell, then the prisoner must also make it to the cell.

With this functional form, as $e \to \infty$, $p \to \alpha$ and when $e = 0$, $p(0) = \alpha - \beta$. Thus, we set $\alpha = 1$ and $\beta = 0.499$. Additionally, we assume the cost function takes the functional form

$$c(e) = \frac{\gamma}{2} e^2. \tag{7}$$

Hence, marginal cost is positive, increasing, and parameterized by γ. For the numerical calculations we set $\gamma = 1000$. Additionally, the benefit to making the correct decision is st at $b = 1000$.

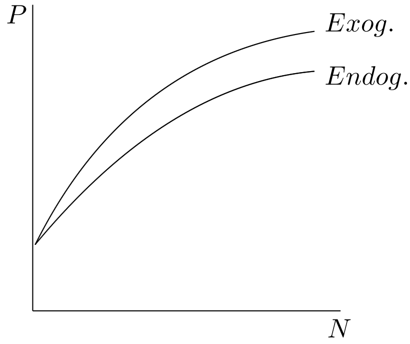
With this setup Table 1 presents the equilibrium level of individual effort, with resulting individual and group probability of making a correct decision, for group sizes of 1, 3, 5, 7, and 9. Additionally, the counterfactual outcome is derived where competence is treated as exogenous. In this calculation, the accuracy if the group size is one is (fixed and) used. We do this because it is, in the model, the accuracy of an individual if there is no free-riding problem. Finally, the last column presents the change in the probability the group makes a mistake from exogenous to endogenous competence.

Table 1: A Numerical Example

Group Size	Effort Level	Individual Comp.	Group (Endog.)	Group (Exog.)	Change in Error Rate
1	0.2967	0.6152	0.6152	0.6152	—
3	0.2136	0.5888	0.9307	0.9430	22.1%
5	0.1943	0.5822	0.9873	0.9916	51.2%
7	0.1869	0.5796	0.9977	0.9988	91.7%
9	0.18378	0.5785	0.9996	0.9998	100.0%

At a committee of nine, the effort exerted by each individual is only 62% of the level if the individual is the sole decision maker. Thus, free riding can be substantial. Even with relatively low individual competence levels, aggregation

Figure 3: Effect of Endogenous Competence



quickly escalates the quality of the committee outcome. This is a commonly noted feature of the Theorem (Congleton, 2007). The difference between endogenous and exogenous competence is nontrivial. The gap in the error rates increases from 22% when there are three in the group to 100% when there are nine. The discrepancy in accuracy, though, mitigates as group size grows, as is suggested by Proposition 3. Figure 3 graphically depicts the effect of endogenous competence on the Condorcet Jury Theorem.

Considering welfare, it is straightforward to verify that for any size of the committee, investments in competence are suboptimal compared to what would have been selected by a planner maximizing aggregate welfare. To see this, consider the first-order condition presented in (5). The marginal benefit for a single individual only considers the impact of the investment on the benefit received from swinging the outcome when there is a tie. It does not include the benefit of the investment to the other $N-1$ individuals. Therefore, the social marginal benefit exceeds the private marginal benefit and, consequently, since $c(e)$ is convex and $p(e)$ is concave, welfare-maximizing effort is higher.

4 Extensions

The main result, that free riding is a concern in group decision making and that it is exacerbated when groups expand, raises the question of how do (or can) institutions be designed to mitigate this shortcoming. An important consideration in the environment presented is that the first-stage investment is not observable. Hence, contracts cannot be formed. Additionally, with observable competence wieighted voting rules would be optimal (Nitzan and Paroush, 1982; Ben-Yashar and Nitzan, 1997) and, in certain circumstances, delegation to an expert may be best. Additionally, a planner, bureaucrat, manager, etc. is unable to design a mechanism to elicit optimal investments, as in Clarke (1971) and Groves (1973). Common institutional arrangements, though, can be expected to interact with endogenous competence. Two are considered here, namely member exit and supermajority rules. We consider these two reasonble extensions to the environment.

4.1 Exit

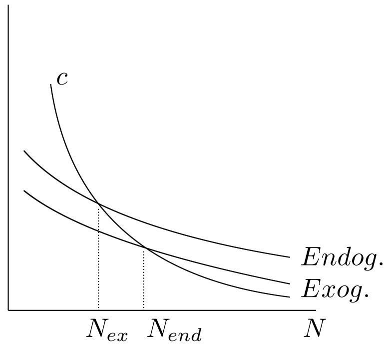
First, if entry into the group creates a disincentive effect on the others on the committee, then it seems reasonable that exit from group will promote improved decision making. Consider the payoff of an individual deciding to exit the committee in a stage-zero choice. To keep the analysis simple, we will continue with the restriction to odd-sized groups. Thus, consider the decision of two individuals deciding whether or not to exit.

To stay on the committee with N individuals, a person's payoff is $P_N b - x(\widehat{e}_N)$ where $\widehat{e}_N$ is, as before, the symmetric equilibrium investment with N members of the committee and P_N is the resulting group competence, from equation (2). Instead, if the two individuals exit, they receive a payoff of $P_{N-2}b$ where P_{N-2} is the group competence in the symmetric equilibrium with $\widehat{e}_{N-2}$ as the effort investment made by each of the $N-2$ individuals on the committee. Consequently, the two will stay on the committee so long as

$$(P_N - P_{N-2})b \geq c(\widehat{e}_N). \tag{8}$$

The cost function is increasing in the effort investment and, as shown in Proposition 2, the investment is decreasing in N. Therefore, the cost function is decreasing in N.

Figure 4: Endogenous Group Size



Consider, first, decision making with exogenous competence. It is straightforward to verify that the competence function is concave. Consequently, $\overline{P}_N - \overline{P}_{N-2}$ is positive and decreasing (where $\overline{P}_N$ is the group competence when accuracy is exogenous). Furthermore, as a consequence of the asymptotic Condorcet Jury Theorem, the difference goes to zero as $N \to \infty$. As a result, the number of individuals who participate in the committee balances the two, equation (8).

Now consider the impact of endogenous competence. It follows from Proposition 2 that $\overline{P}_N > \widehat{P}_N \; \forall N$. Furthermore, it follows from Proposition 3, that $\widehat{P}_N - \widehat{P}_{N-2} \to \overline{P}_N - \overline{P}_{N-2} \to 0$ as $N \to \infty$. Therefore, so long as $\widehat{P}_N - \widehat{P}_N > \overline{P}_N - \overline{P}_{N-2} \; \forall N$, then the committee size decreases. Figure 4 graphically illustrates.

This represents a further limitation on the Condorcet Jury Theorem. The main result (Proposition 2) is that the marginal benefit to group size is less, due to the public goods nature of intelligent voting. The secondary effect is the observation that the marginal benefit of the individual to participating on the committee is mitigated, so that the committee will endogenously shrink. The change in the equilibrium group size is not due to costs directly, but rather driven by the avoidance of free riding by others.

4.2 Supermajority Rules

Second, free riding occurs in this framework because in large groups there is only a small chance that one's vote is needed to reach the majority threshold required to implement the correct policy. When many people, who are each reasonably competent, are voting, then the probability they collectively reach the correct decision is quite high. A person's effort has little effect on the probability of swinging the vote.

This can be expected to change if a voting rule other than simple majority voting was in place. In dichotomous choice problems, the natural extension, then, is to consider supermajority voting rules, also known as a k-majority rule (Dougherty and Edward, 2012); see Dougherty (2015) for a discussion. While the accuracy of the other individuals might, collectively, be high, if the threshold needed to implement an outcome is substantial, the importance of a person's vote improves.

Supermajority rules have been evaluated in a Condorcet Jury Theorem environment. Fey (2003) verifies the applicability of the asymptotic result in such an environment. Nitzan and Paroush (1984) evaluate their ability to improve group decision making by reducing the chance of implementing the incorrect outcome when competence is heterogeneous. Dougherty, Edward, and Ragan (2015) introduce judgment errors to evaluate optimal supermajority rules. Therefore, supermajority rules are a natural place to consider the impact of endogenous competence.

Thus, consider an extension to the environment where a non-decisive supermajority voting rule is in place. A non-decisive rule does not require that one of the two options is selected. Rather, for an outcome to be chosen k votes is needed. If neither option receives k votes, then neither is implemented and, in the framework presented here, there is a zero probability of realizing the benefit b. For it to be a supermajority vote, k must exceed $\frac{N+1}{2}$. Unanimity, for example, is $k = N$. A $\frac{3}{4}$s voting rule has $k = 0.75N$ (or the nearest integer greater than $0.75N$).

In this extension, denote the probability the group implements the correct outcome as K_N. Hence, it follows that if a symmetric investment profile E is selected,

$$K_N(E) = \sum_{x=k}^{N} \frac{N!}{x!(N-x)!} p^x (1-p)^{N-x}. \tag{9}$$

Notice that, holding fixed e for each individual, $K_N = P_N - R$ where R is the expression in the right-hand-side of (9) using values of x summed over $x = \frac{N+1}{2}$ to $x = k-1$ (i.e., the "remainder"). Since R is non-negative, $P_N \geq K_N \ \forall N$ for any symmetric profile E.[9]

Relatedly, the probability of a tie is greater, for each N. From (3) a tie under simple majority requires $\frac{N-1}{2}$ to be incorrect. A tie in a supermajority vote requires only $N-k$ incorrect votes. Thus, the chance of being able to break a tie towards implementing the correct option is higher when a supermajority rule is in place. Similarly, the probability of breaking a tie in favor of the incorrect option is less with a supermajority voting rule in place.[10] Therefore, it follows from (5) that the marginal benefit to competence is greater with a supermajority rule. Consequently, individual competence is higher with a supermajority threshold and, holding fixed N is increasing in the voting threshold.

Proposition 4. *The symmetric equilibrium investment is strictly increasing in the (non-decisive) supermajority threshold k.*

Proof. An individual's optimal e_i solves (5) where T_i^N is replaced with $\tilde{T}_i^N$ defined as $\frac{(N-1)!}{k!((N-1)-k)!} p^k (1-p)^{k-\frac{N-1}{2}}$. Consider, first, $k = \frac{N-1}{2} + 1$. It follows immediately that $\tilde{T}_i^N = \left(\frac{N-1}{N+1}\right)\left(\frac{p}{1-p}\right) T_i^N$. Since $p > \frac{1}{2N} + \frac{1}{2}$, $\tilde{T}_i^N > T_i^N$. Following this reasoning, $\tilde{T}_i^N$ is increasing in k. Since $p(e)$ is strictly concave and $c(e)$ is strictly convex, it follows that $\widehat{e}$ is decreasing in k. □

Therefore, the use of a supermajority rule mitigates the free-riding problem.

Concerning the effect of group size on competence, the result in Proposition 2 extends to supermajority voting rules. For a fixed, non-decisive supermajority

[9] The two equal if and only if k is one vote greater than one-half, $k = \frac{N+1}{2} + 1$.

[10] To simplify the proof the assumption $p > \frac{1}{2N} + \frac{1}{2}$ for a fixed e is employed. Allowing p to equal $\underline{p}$ provides a sufficient (but not necessary) condition.

voting rule, an increase in group size reduces the probability of a tie amongst the other $N-1$ voters, which disincentivizes effort as the marginal benefit of it reduces. Thus, the marginal benefit to expanded group size is mitigated.

Proposition 5. *For any (non-decisive) supermajority voting rule, the symmetric equilibrium investment is strictly decreasing in N.*

Proof. The proof is a straightforward extension of the proof of Proposition 2. The tie probability $\tilde{T}_i^N$ is decreasing in N. Since $p(e)$ is strictly concave and $c(e)$ is strictly convex, an increase in N decreases $\widehat{e}_i$. □

This highlights another impact of the use of supermajority voting rules. They are frequently employed to entrench the status quo. For example, they are needed to amend a constitution, impeach an elected or appointed government official (e.g. the President or a judge), or remove a corporate officer during an takeover attempt (Mahoney, Sundaramurthy, and Mahoney, 1996). Supermajority rules also encourage investments in competence. This is a previously undocumented result.

5 Conclusion

We explore the endogeneity of competence in a group decision-making context. We show that the improvement in the accuracy of a decision with increases in the size of the group is diminished. The decision to invest in competence is considered, illustrating that it suffers from a free-riding problem. In other words, intelligence is a public good. Our main result is that as the size of the committee grows, the free-riding problem becomes more severe as individuals each invest less in their competence. This counteracts the information aggregation benefit. Thus, the benefits to increased group size are not as great. Coupled with costs to group size, committees need not be that large.

We are able to resurrect though, an important component of the Condorcet Jury Theorem. We establish that even in the presence of free riding, the accuracy of the group's decision goes to one as the group size increases to infinity.

This has been referred to as the asymptotic Condorcet Jury Theorem (McCannon, 2015). Thus, while intermediate group sizes may suffer from free riding, sufficiently large committees can still be expected to make good decisions.

There are a number of related issues worth exploring. For example, under what conditions does this result lead to delegation to a single expert as being optimal? Relatedly, Ben-Yashar, Koh, and Nitzan (2012) consider decision making within committees where members may have specialization, but do not consider endogenous specialization. Also, the interaction between endogenous competence and interdependent information needs to be explored. It is possible that the free-riding problem may mitigate the concern for informational cascades when information is correlated across players. Finally, polychotomous choice decision making is not well understood (Ben-Yashar and Paroush, 2001). See Hummel (2010) for a recent advance. If there are numerous options, committee members could, for example, coordinate their effort investments by delegating effort to differing options. The endogeneity of competence in these environments warrants future exploration.

The result presented here provides a guide for appreciating why groups outperform individuals, but delegation to committees can outperform decision making by the universal set of people. It complements the transaction cost argument for group decision making introduced by Buchanan and Tullock (1962). Consequently, future work could further consider the relationship between endogenous competence and voting rules other than simple majority.

Finally, theoretical results are presented. It is an open question whether predictions of the model match real-world behavior. Miller (1996), for example, presents empirical evidence that the Condorcet Jury Theorem is valid. Experimental studies of public goods contributions document a warm-glow effect (Andreoni, 1995) where individuals may have preferences for an avoidance of free riding. These preferences can be enhanced with face-to-face communication (Cason and Khan, 1999) and by being a leader in a sequential-giving environment (McCannon, 2016). The effect of endogeneity of competence on group decision making, though, may need empirical verification though.

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