Abstract

I introduce patents into a general equilibrium model of innovation, where innovators choose between creating a new product market and competing in an existing market. Patent holders demand royalties from sequential innovators, but are constrained by the ability of innovators to work around patents. I show patent protection acts as a net tax on sequential innovators, reducing both competition and productivity growth. Calibrated to match moments from U.S. data, the model predicts that eliminating patent protection in the U.S. would generate a 23% increase in steady-state productivity growth as well as an increase in welfare equivalent to that from a 16% increase in annual consumption. I test several implications of the model using both U.S. and cross-country data. Consistent with the model, the data suggests an increase in the strength of patent protection reduces both productivity growth and the average quality of innovations.

JEL: O1 O4

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1 Introduction

The last decade has seen renewed interest in two strands of the innovation literature, and in more recent years the two have started to interact. Aghion et al. (2005) renewed interest in the relationship between competition and innovation, motivating a wealth of both empirical and theoretical studies exploring the nuances of this relationship and its policy implications. A series of papers (as well as a book) by Boldrin and Levine (2002, 2005, 2008a, 2008b, 2013) questioning the need for and efficacy of patent rights have at the same time inspired a growing number of empirical and theoretical studies investigating the relationship between patent protection and innovation.

Boldrin and Levine (2008b) argue that the large and still growing number of studies finding a positive relationship between competition and innovation is prima facie evidence against the presumed need for patent rights to encourage innovation. Meanwhile Aghion, Howitt, and Prantl (2013b) present evidence that the pro-innovation effects of product-market deregulation are increasing in the strength of patent rights, and argue Boldrin and Levine may be overlooking some potential benefits of patent protection. Given the inability of current workhorse models of R&D-based growth to generate a positive link between competition and innovation, they are of limited value in evaluating Boldrin and Levine’s claim. More generally, the models currently used to discuss patent policy are of limited use in thinking about ‘competition’, as traditionally defined (the number of actual or potential competitors). In extensions of Grossman and Helpman (1991) the number of firms is indeterminate, while in IO-style models the number of firms is fixed at two.\(^1\)

In this paper I bring together ideas from both the competition and innovation literature and the patent rights literature to build a model which captures many of the key facts

\(^1\)In Grossman and Helpman (1991) only the aggregate level of research in a market is pinned down. While only one firm produces at any point in time, the number of firms doing research is indeterminate. Extensions of Grossman and Helpman include O’Donoghue and Zweimüller (2004), Futagami and Iwaisako (2007), and Acemoglu and Akgül (2012). Examples of IO-style models include Bessen and Maskin (2009) and Aghion, Howitt, and Prantl (2013a).
about competition, innovation, and the effects of patent rights documented in empirical studies. In the model, product-market deregulation results in more competition, innovation, and productivity growth (Bottasso and Sembenelli (2001), Nicoletti and Scarpetta (2003), Griffith, Harrison, and Simpson (2010)); more patent protection leads to less innovation and growth (Boldrin and Levine (2008b, 2013), Lerner (2009), Moser and Voena (2012), Moser (2013));\textsuperscript{2} and product-market deregulation has a larger impact when patent rights are strong (Aghion, Howitt, and Prantl (2013b)). The model is ambiguous with respect to the effect of patent protection on welfare, as the lower growth caused by stronger protection is accompanied by a higher consumption share of output and an increase in the variety of products available. To estimate the welfare costs (or benefits) of patent protection in practice, I calibrate the model using U.S. manufacturing data. The calibrated model predicts that the elimination of patent rights in the U.S. would generate a 23% increase in productivity growth and a 21% drop in the number of product varieties, resulting in a net increase in steady-state welfare equivalent to that from a 16% increase in annual consumption.

An additional implication of the model is that stronger patent protection should lead to a decrease in the average quality of innovations. Using data on U.S. patent applications originating in foreign countries, I provide new evidence that the average number of citations per patent (a proxy for the average quality of innovations) is indeed driven lower when patent protection is made stronger. In addition I provide corroborating evidence of the previously documented negative effects of patent protection on overall patenting activity and productivity growth.

The model economy developed in this paper is similar to Chu, Cozzi, and Galli (2012) in that the number of differentiated products (varieties) within an industry is endogenous, and firms compete within each product market. But while in Chu, Cozzi, and Galli the number of competitors in each market is indeterminate, competition in the present paper

\textsuperscript{2}Boldrin and Levine (2008b) provide a review of the large empirical literature studying the connection between patent rights and innovation, as do Gallini (2002) and Jaffe (2000).
is modeled as in Bento (2013). Each period, firms compete within a product market by introducing improved versions of an existing product. These sequential innovators conduct research to increase the magnitude of their respective improvements, but are ex ante uncertain about the optimal direction of a quality improvement. Once the cost of research and the cost of introducing a product into a market are sunk, firms discover the value of their own improvement and those of their competitors. By allowing for the ‘best’ innovation to capture a market I create a link between the number of innovations and the level of quality growth within a product market. This mechanism captures the insight of Hayek (2002) that competition allows a thousand ideas to bloom (to mix metaphors), then works to discover the best among them.

A key implication of the model is that patent protection acts as a tax on sequential innovation and competition. That this must be the case follows directly from the fact that the higher costs incurred by sequential innovators improving an existing product today (in the form of royalty payments to patent holders) are larger than the expected discounted revenue from royalty payments in the future, a point made by Chu, Cozzi, and Galli (2012). As these higher costs are effectively transfers from sequential innovators to creators of new differentiated products (who I assume do not infringe on previous patents), stronger patent protection involves a reallocation of resources away from growth to maintain a greater variety of products. As a result, patent protection is welfare-improving only if growth is otherwise ‘too high’.

This paper contributes to the theoretical literatures on competition, innovation, and patent policy, as well as the empirical literature examining the efficacy of patents in encouraging innovation and growth. Aghion et al. (2005) develop a model where some measure of competition can be positively correlated with innovation and growth, but do not allow for the free entry of firms. Boldrin and Levine (2008a, 2009) develop models in which innovation occurs (and can even be optimal) under perfect competition, but are unable to speak to the
relationship between innovation and differing levels of competition.

Scotchmer (1991) analyzes optimal patent policy when innovation is sequential, as do a large number of subsequent papers like O’Donoghue and Zweimüller (2004), Bessen and Maskin (2009), and Acemoglu and Akcigit (2012). In each of these papers the number of innovators is either fixed at two or indeterminate, while the model developed here features an endogenous number of sequential innovators.

In the next section I describe the model, characterize the competitive equilibrium, and discuss its key implications. In Section 3 I calibrate the model and show that eliminating patent protection in the U.S. would increase welfare. In Section 4 I provide empirical evidence supporting the implications of the model with respect to the relationships between patent protection, innovation, and productivity growth. The final section concludes.

2 The Model

Consider an economy in which a final good is produced using a variety of inputs from a representative intermediate industry. Intermediate firms (hereafter refered to as ‘firms’) produce these inputs one-for-one using labor. As such, output per worker in this economy is equivalent to aggregate total factor productivity. The final good can be used for consumption or innovation, and will also act as the numéraire. There are a large number of potential innovating firms, any of which can choose to introduce either a new product (thereby creating a new product market), or an improved version of an existing product. Each product market faces an exogenous probability of destruction each period, so new product markets continue to be created in steady state. I study the stationary competitive equilibrium of the economy along a balanced growth path, in which firms take the economy-wide wage, growth rate, and interest rate as given, and free entry ensures zero expected profits for all entrants. I begin by describing the environment in more detail.
2.1 Environment

There is a representative consumer who supplies one unit of labor to intermediate firms. The consumer only values consumption \((C)\) and has a constant discount factor \(\beta \in (0, 1)\). Preferences over the stream of consumption in each period are described by the following log-utility function:

\[
\sum_{t=0}^{\infty} \beta^t \log(C_t).
\]

The market for final output is perfectly competitive, with a representative firm using inputs from a representative intermediate industry to produce output according to the following production function:\(^3\)

\[
Y = \left( \int_0^M A_m^{1-\alpha} y_m^\alpha dm \right)^{\frac{1}{1-\alpha}},
\]

where \(m\) indexes a continuum of product markets of measure \(M\), \(y_m\) is the quantity of input \(m\) used, \(A_m\) is the quality of input \(m\) (or alternatively, the productivity of input \(m\) in final good production), and \(\frac{1}{1-\alpha}\) is the constant elasticity of substitution between differentiated products. Under the above assumptions, \(Y\) is equivalent to both output per worker and aggregate total factor productivity.

I assume different products within a product market are perfectly substitutable. I further assume some epsilon fixed operating cost and that the firm with the 1st-best quality \(A = A_{[1]}\) can commit to charging a limit price if any rivals attempt to produce, thus ensuring only the best firm in a market produces in equilibrium.\(^4\)

Finally, I assume product markets are destroyed with probability \(\delta\) each period after production takes place but before any costs associated with innovation are incurred.

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\(^3\)Throughout the paper, I omit the time subscript unless clarity requires it.

\(^4\)In Bento (2013) I consider a market with no operating cost, where the best firm charges a price equal to the marginal cost of its closest rival (adjusted for quality) in equilibrium. Incorporating these variable markups in the present model would complicate the analysis without contributing any insights to those already discussed in Bento (2013).
2.2 Innovation

In each period, after both intermediate inputs and the final good have been produced, firms can choose to undertake the investment necessary to create and introduce a new or improved product. I first describe the innovative process for firms introducing an improved product into an existing product market.

2.2.1 Existing Product Market

Denote the quality of an incumbent’s product in period $t$ as $A_{[1],m,t}$, where $[1]$ indicates that the incumbent has the 1st-best quality in period $t$ in market $m$. Any firm-$i$ wishing to incur the cost of introducing an improved version of product $m$ in the subsequent period receives a quality equal to $A_{i,m,t+1} = A_{[1],m,t} \cdot h_{i,m,t+1}$, where $h_i$ is a random variable bounded by 1 and $1 + x_{i,m,t}$, and $x_i$ is chosen by the firm. I assume the cost of $x_{i,m,t}$ is $\Psi_{m,t} \cdot \frac{x_{i,m,t}}{\theta}$, where $\theta > 1$ is the elasticity of research expenditure with respect to the upper bound chosen by the firm. Before realizing the value of its draw (as well as the draws of other firms) the innovating firm must also incur a fixed cost of introducing its product equal to $\Psi_{m,t} \cdot c_{S}^{F}$, where the $S$-superscript refers to sequential innovation. One can think of the level of research as determining the magnitude of a quality improvement, and the randomness of the draw reflecting a firm’s ex ante uncertainty about which characteristics of the product to improve upon.\footnote{Throughout the paper, I will often refer to a firm’s choice of $x$ as that firm’s ‘level of research’.

5 Only once all of these costs are sunk do firms learn their own and each other’s quality. By multiplying these costs by $\Psi_{m,t} \equiv Y_{t} \cdot \frac{A_{[1],m,t}}{E(A_{[1],t})}$, I ensure innovating firms will face the same decision each period in steady state.

2.2.2 New Product Market

Any firm choosing to introduce a new product (thus creating a new product market) in period $t + 1$ must incur a fixed cost in period $t$ equal to $Y_{t} \cdot c_{0}^{F}$, where the 0-subscript reflects
the fact that a new market is being created. I assume such a firm receives a quality equal to the industry average, $E(A_{i[t_1,t+1]}).$ For an equilibrium with both new-product and sequential innovation to exist, $c^0_F$ must be sufficiently large relative to $c^S_F$.

2.3 Patents

Upon introducing a product into a market, a firm can costlessly obtain a patent. I assume firms introducing new products never infringe on existing patents, while sequential innovators always do. I further assume patent holders commit ex ante to charging a royalty fee to subsequent infringers equal to some fixed fraction $\rho$ of both operating profits and any future royalty revenue the infringer may receive. I assume an innovator who pays these royalty fees to a patent holder takes ownership of the previous patent, so that a patent holder producing in period $t$ only receives royalty payments from an innovator producing in period $t+1$ (who then passes along payments from the next innovator, etc...). At the same time I assume a patent holder’s choice of $\rho$ is constrained by the ability of sequential innovators to work around a patent (and thus not infringe) by incurring larger innovation costs, equal to $\hat{\rho} \cdot \Psi_m (c^S_F + \frac{c^0_S}{\hat{\rho}})$; $\hat{\rho} > 1$. I interpret $\hat{\rho}$ as a measure of the strength of patent protection. But given the royalty fee $\rho$ will be a strictly increasing function of $\hat{\rho}$, from this point on I simply use $\rho$ as my measure of patent protection.

2.4 Competitive Equilibrium

I focus on the stationary competitive equilibrium of the model. In such an equilibrium, the interest rate $r$ and growth rate $g$ are constant, as is the measure of product markets $M$, and the wage $w$ grows at the same rate as total output. In addition, the assumptions made about the environment above will ensure both a constant number of sequential innovators $N$ per market in each period and a constant level of research per (sequential) innovation $x$.

6The model can easily be extended to allow for a research decision by new-product innovators, but such an extension complicates the model without contributing any interesting implications.
I begin by describing the decision problems of each agent, and then define and solve for the stationary equilibrium.

### 2.4.1 Consumer

In each period the consumer chooses both consumption and savings, and the only vehicle for savings is the purchase of equity in innovating intermediate firms, earning a rate of return of \( r \).\(^7\) The consumer’s problem is therefore to choose consumption \( C \) and savings \( S \) in each period \( t' \), given \( w, r, \) and \( g \), to maximize;

\[
\sum_{t=t'}^{\infty} \beta^t \log(C_t), \quad \text{s.t. } C_t + S_t \leq w_t(1 + g)^{t-t'} + S_{t-1}(1 + r).
\]

The first order conditions for this problem imply the following interest rate;

\[
r = \frac{1 + g}{\beta} - 1.
\]

### 2.4.2 Final-Good Producer

In each period, the final-good producer takes the prices of all intermediate inputs as given, and demands inputs from each intermediate firm to maximize profits;

\[
Y - \int_0^M P_m y_m dm,
\]

where \( Y \equiv \left( \int_0^M A_{[1],m}^{1-\alpha} y_m^\alpha dm \right)^{\frac{1}{\alpha}} \) and \( P_m \) is the price of input \( m \). The first order conditions for the final-good firm’s problem imply the following inverted demand function for each intermediate input;\(^8\)

\[
P_m = Y^{1-\alpha} A_{[1],m}^{1-\alpha} y_m^{\alpha-1}.
\]

\(^7\)I take it as given that the consumer will diversify across all innovating firms, as they will all share the same expected value before innovating.

\(^8\)I have taken it for granted here that only one intermediate firm will produce in each product market.
2.4.3 Intermediate Firms

In each period, once the quality of each innovator is realized, all but the highest-quality firm in each product market will choose not to produce (given the assumptions above). The remaining firms face the downward-sloping demand curves implied by the final-good firm’s problem above, and demand labor $y_m$ given the wage $w$, to maximize operating profits;

$$\pi_m = P_m y_m - w y_m.$$ 

First order conditions imply an optimal price equal to;

$$P_m = \frac{w}{\alpha},$$

and optimal output equal to;

$$y_m = Y A_{[1],m} \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}}.$$ 

Together, these imply a firm with quality $A_m$ earns operating profits equal to;

$$\pi_m = y A_{[1],m} (1 - \alpha) \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}.$$ 

A sequential innovator that realizes the highest quality $A$ in market $m$ will earn a fraction $1 - \rho$ of the above operating profits (with the remainder transfered to the previous patent holder), as well as a fraction $\rho (1 - \rho)$ of the operating profits and royalty revenue of the following period’s best firm (again, with $\rho^2$ being transfered to the previous patent holder). The value of introducing an improved product into an existing market $m$ for some firm-$i$ in period $t$ can now be expressed as;

$$V_{i,m,t-1} = -\Psi_{m,t-1} \left(\frac{\theta}{c_F} + \frac{x_{i,m,t-1}^S}{\theta}\right).$$
\[ (+ \text{Prob}(A_{i,m,t} = A_{[1],m,t}) \frac{(1 - \rho)(1 - \alpha)\alpha^{\frac{1}{1-\alpha}}}{1 + r} \sum_{t'=t}^{\infty} E(A_{[1],m,t'}|A_{i,m,t} = A_{[1],m,t}) \frac{(1 - \delta)^{t'-t} \rho t' - t Y_{t'}}{(1 + r)^{t'-t} w_{t'}^{1 - \alpha}}. \]

By taking for granted that the wage grows at the same rate as final output, and using \( A_{i,m,t} = A_{[1],m,t-1} \cdot \rho_{i,m,t} \) and \( 1 + r = (1 + g)/\beta \), the above can be expressed more compactly as:

\[ V_{i,m,t-1} = -\Psi_{m,t-1} \left( c_F^S + \frac{x_{i,m,t-1}^0}{\theta} \right) + \text{Prob}(h_{i,m,t} = h_{[1],m,t}) E(h_{[1],m,t}^0) \frac{A_{[1],m,t-1}(1 - \rho)\beta(1 - \alpha)Y_{t-1}}{\left(1 + g\right)^{\frac{1}{1-\alpha}} - \rho \beta(1 - \delta)E(h_{[1],m,t'})} \left(\frac{\alpha}{w_{t-1}}\right)^{\frac{1}{1-\alpha}}, \]

where \( c_F^S \) is the fixed cost of introducing an improved product, \( x_i \) is firm-\( i \)'s level of research, \( h_i \) is the realization of firm-\( i \)'s random draw, \( h_{[1],t} \) is the best draw, \( A_{[1],t-1} \) is the best quality in period \( t-1 \), \( E(h_{[1],t'}) \) is the expected value of the best draw in future periods (constant in equilibrium), \( \delta \) is the probability of product-market destruction, and \( \Psi_{m,t-1} \equiv Y_{t-1} \frac{A_{[1],m,t-1}}{E(A_{[1],t-1})} \).

Given a firm-\( i \)'s decision to introduce an improved product in market \( m \), it will choose its level of research \( x_i \) to maximize expected discounted profits, resulting in the following optimal research condition:

\[ \frac{\partial}{\partial x_{i,m,t-1}} V_{i,m,t-1} = 0, \]

where \( \text{Prob}(h_i = h_{[1]}) E(h_{[1]}^0) \) depends on firm-\( i \)'s level of research \( x_i \), every other firm's level of research \( x_{-i} \), and the number of products being introduced \( N \).

An innovator choosing to introduce a new product, thus creating a new product market, pays no royalty and faces no uncertainty, but must incur a fixed cost of \( c_F^0 \cdot Y \). The value of introducing a new product is therefore:

\[ V_{0,t-1} = -Y_{t-1} \cdot c_F^0 + \frac{(1 - \alpha)\alpha^{\frac{1}{1-\alpha}}}{1 + r} \sum_{t'=t}^{\infty} E(A_{[1],t'}) \frac{(1 - \delta)^{t'-t} \rho t' - t Y_{t'}}{(1 + r)^{t'-t} w_{t'}^{1 - \alpha}}, \]

or

\[ V_{0,t-1} = -Y_{t-1} \cdot c_F^0 + E(h_{[1],t}) \frac{E(A_{[1],t-1})\beta(1 - \alpha)Y_{t-1}}{\left(1 + g\right)^{\frac{1}{1-\alpha}} - \rho \beta(1 - \delta)E(h_{[1],t'})} \left(\frac{\alpha}{w_{t-1}}\right)^{\frac{1}{1-\alpha}}. \]
Finally, note that firms will never choose to introduce more than one improved product into an existing market in the same period. Introducing a second product would reduce the expected value of introducing the first, so that a firm will always prefer to introduce a second product into a differentiated market. This implies the number of firms in an industry is indeterminate, although the number of firms in each product market is pinned down in equilibrium.\footnote{The model could presumably be extended to pin down the number of firms and number of products per firm in an industry as in Bernard, Redding, and Schott (2010), but such an extension is beyond the scope of the present paper.}

\subsection*{2.4.4 Stationary Competitive Equilibrium}

Here I take for granted that all sequential innovators face the same problem and all new-product creators also face the same problem. A stationary competitive equilibrium is a constant number of sequential innovators per market $N$, level of research per innovation $x$, measure of product markets $M$, and markup over marginal cost $P/w$, as well as a constant growth rate $g$ of intermediate prices $P$, final-good output $Y$, and the wage rate $w$, such that:

(i) Consumer Optimization: $r = \frac{1+g}{\beta} - 1$

(ii) Final-Good Firm Optimization: $P_m = \left( \frac{YA_{[1],m}}{y_m} \right)^{1-\alpha}$

(iii) Intermediate Producer Optimization: $\frac{P_m}{w} = \frac{1}{\alpha}$

(iv) Sequential Innovator Optimization: $\frac{\partial}{\partial x_i} V_{i,m} = 0$

(v) Free Entry: $V_{i,m} = 0$ and $V_0 = 0$

(vi) Market Clearing (Goods): $Y = \left( \int_0^M A_{[1],m} y_m^\alpha dm \right)^{\frac{1}{\alpha}}$

(vii) Market Clearing (Labor): $1 = \int_0^M y_m dm$

where conditions (ii) through (v) are understood to hold for all $m \in M$.

To solve for the stationary equilibrium I start with the market clearing conditions and substitute each producer’s optimal output $y_m$ to get:

$$Y = YM^{\frac{1}{\alpha}} E(A_{[1]})^{\frac{1}{\alpha}} \left( \frac{\alpha}{w} \right)^{\frac{1}{\alpha}}$$

and $1 = YME(A_{[1]}) \left( \frac{\alpha}{w} \right)^{\frac{1}{\alpha}}$. 

which can be solved for final-good output and the wage, each as functions of the average quality of producers $E(A_{[1]})$ and the measure of product markets $M$:

$$Y = M^{\frac{1-\alpha}{\alpha}} E(A_{[1]})^{\frac{1-\alpha}{\alpha}}, \text{ and } w = \alpha Y.$$ 

As the measure of product markets is constant, the growth rate $g$ can be derived by using $A_{[1],m,t} = A_{[1],m,t-1} \cdot h_{[1],m,t}$ to get:

$$\frac{Y_t}{Y_{t-1}} = 1 + g = E(h_{[1]})^{\frac{1-\alpha}{\alpha}}.$$

The equilibrium number of sequential innovators per market $N$, level of research per innovation $x$, and measure of product markets $M$ can now be characterized using the free entry and optimal research conditions above;

- free entry (new markets): $c_F^0 = \frac{\beta(1-\alpha)}{M[1-\rho\beta(1-\delta)]}$
- free entry (existing markets): $c_F^\theta + \frac{x^\theta}{\theta} = \frac{(1-\rho)\beta(1-\alpha)}{NM[1-\rho\beta(1-\delta)]}$
- optimal research: $x^{\theta-1} = \frac{(1-\rho)\beta(1-\alpha)}{E(h_{[1]})M[1-\rho\beta(1-\delta)]} \cdot \frac{\partial}{\partial x_i} \text{Prob}(h_i = h_{[1]}E(h_{[1]})^1)_{x_i=x_{i-1}}$

Given some distribution for $h_i$ (conditional on $x_i$), the random variable determining each sequential innovator’s quality, all other variables are functions of $N$, $x$, and $M$.

### 2.5 Results

Here I examine how the stationary equilibrium of the economy depends on the strength of patent protection and the level of product-market regulation. I begin by considering equilibria associated with different levels of patent protection, measured here by the fraction $\rho$ of profits which infringers must pay in royalty fees. Consider first the free entry condition
for new-product creators, above. It is immediately obvious that the denominator on the right-hand side must remain constant across different values of $\rho$. This implies that the measure of product markets $M$ must be increasing in patent protection. New-product creators never infringe on previous patents, so any increase in protection simply increases the present value of future royalty revenue, holding $M$ constant. But with free entry, the measure of firms creating new product markets each period increases until the increase in revenue is completely dissipated.

Now consider the free entry and optimal research conditions for sequential innovators. Given that $M[1 - \rho \beta (1 - \delta)]$ is constant in $\rho$, the net effect of $\rho$ on sequential innovators is equivalent to that of a simple tax on profits, reducing both the number of innovations $N$ and the level of research per innovation $x$. Since the growth rate of the economy is increasing in both $N$ and $x$, it must be the case that growth is decreasing in patent protection.\(^{10}\) Note that this is not simply a result of the increase in the measure of product markets $M$, which itself lowers the profits of innovators. Consider for a moment increasing $\rho$ while holding $M$ fixed. It is still the case that the profits of sequential innovators are decreasing in $\rho$, which continues to act as a tax on both the number of innovations and the level of research per innovation. The increase in $M$ simply magnifies the negative impact on growth. That growth is decreasing in patent protection is broadly consistent with the empirical evidence discussed in the introduction, and in Section 4 I provide some additional evidence in support of this conclusion. To my knowledge there has been no test of the hypothesis that the average quality of innovations decreases with stronger patent protection. In Section 4 I provide evidence in support of this hypothesis.

Before discussing the effect of patent protection on welfare, it is useful to manipulate the three equilibrium conditions to obtain the investment rate of the economy $I$, the share

\(^{10}\)From above, the growth rate is an increasing function of $E(h_{11})$. That the best of $N$ draws is increasing in $N$ (conditional on $x$) should be obvious. In Appendix A.1 I show $E(h_{11})$ must be increasing in the level of research $x$ (conditional on $N$).
of output devoted to sequential innovation in each existing market $N \left( c^S_F + \frac{x^\theta}{\theta} \right)$, and the relationship between the number of innovations $N$ and the level of research $x$;

$$I = M \left[ \delta c^0_F + (1 - \delta)N \left( c^S_F + \frac{x^\theta}{\theta} \right) \right] = \frac{\beta(1 - \alpha)[1 - \rho(1 - \delta)]}{1 - \rho \beta (1 - \delta)}$$

$$N \left( c^S_F + \frac{x^\theta}{\theta} \right) = (1 - \rho)c^0_F$$

$$x^\theta = \frac{\theta N x^S_F \cdot \zeta(x_i, x_{-i}, N)}{\theta E(h_{[1]}) - N x \cdot \zeta(x_i, x_{-i}, N)}$$

where $\zeta(x_i, x_{-i}, N) \equiv \frac{\partial}{\partial x_i} \text{Prob}(h_i = h_{[1]}|E(h_{[1]})| x_i = x_{-i} = x)$. The first two equations show both total investment and investment in sequential innovation are decreasing in the strength of patent protection $\rho$. The last equation shows that while patent protection decreases investment in sequential innovation, it does nothing to change the relationship between $N$ and $x$.

In Appendix A.3 I show a constrained social planner would choose an investment rate equal to $1 - \alpha$.\(^{11}\) Equilibrium investment is therefore always lower than optimal, even given the inefficient relationship between the number of sequential innovations per existing market and the level of research per innovation. This all implies that in equilibrium it will never be the case that both the growth rate and the variety of products (which increases the level of productivity) are too high. Patent protection can therefore improve welfare only if the share of investment otherwise allocated to sequential innovation is so high that a relatively small increase in the resources allocated to creating new product markets increases welfare enough to offset a large decrease in resources devoted to sequential innovation. Note this is more likely in an economy with a low discount factor $\beta$, since the optimal growth rate is increasing in $\beta$ while the equilibrium growth rate is independent of $\beta$.

To examine the effect of product-market regulation in the model, I interpret regulation

\(^{11}\)The planner is constrained to maintain the same relationship between $N$ and $x$ shown above.
as the imposition of higher costs on firms attempting to compete with an incumbent in an existing market. Consider the following free entry and optimal research conditions in an existing market where entrants must pay some multiple $\tau > 1$ of the normal costs of innovation, due to regulation;

**free entry (existing markets):**

$$\tau \left( c_F^S + \frac{x^\theta}{\theta} \right) = \frac{(1 - \rho)\beta(1 - \alpha)}{NM[1 - \rho\beta(1 - \delta)]}$$

**optimal research:**

$$\tau x^{\theta - 1} = \frac{(1 - \rho)\beta(1 - \alpha)}{E(h_{[1]}|M[1 - \rho\beta(1 - \delta)])} \cdot \zeta(x_i, x_{-i}, N)$$

where $\zeta(x_i, x_{-i}, N) \equiv \frac{\partial}{\partial x_i} \text{Prob}(h_i = h_{[1]}|E(h_{[1]}|\cdot)) \big|_{x_i = x_{-i} = x}$. Clearly the effects of an increase in the burden of product-market regulation $\tau$ on sequential innovation are identical to those of an increase in patent protection. An increase in $\tau$ leads to fewer firms competing in each market, less research per firm, and lower growth. The only difference between increasing $\tau$ and increasing $\rho$ is that the greater costs incurred by entrants as a result of regulation are not transferred to previous patent holders, which implies the measure of product markets $M$ is independent of $\tau$. That the level of competition (proxied here by the number of competitors) and productivity growth both increase with product-market deregulation has become well established in recent years through both case studies and more systematic empirical studies, as discussed in the introduction.

To examine the interaction between product-market regulation and patent protection, again consider the free entry condition for existing markets. Note that $N \left( c_F^S + \frac{x^\theta}{\theta} \right)$ is unit elastic with respect to $\frac{(1 - \rho)}{\tau}$, so that any increase in the effective tax on profits will cause a corresponding decrease in $N \left( c_F^S + \frac{x^\theta}{\theta} \right)$. Given that both the number of innovations $N$ and the level of research per innovation $x$ decrease with an increase in $\rho$ or $\tau$, it must be the case that $N \cdot c_F^S$ falls slower than overall spending while $N \cdot \frac{x^\theta}{\theta}$ (research intensity in an existing market) falls faster than overall spending. This implies that as the effective tax on profits increases, research intensity becomes a smaller and smaller portion of overall spending.
on innovation. This, in turn, implies that the elasticity of research intensity with respect to the tax is getting larger as the tax increases. All else equal, then, research intensity in an economy with strong patent protection (high $\rho$) will be a smaller portion of total spending on sequential innovation than in an economy with weak protection. This means product-market deregulation (a drop in $\tau$) should have a larger effect on research intensity in economies with strong patent protection. This result is consistent with the findings reported in Aghion, Howitt, and Prantl (2013), though the authors interpret their results in a very different way. While Aghion, Howitt, and Prantl argue their results are evidence of a previously undiscovered benefit of patent protection, the present model suggests that countries with high patent protection are simply deregulating from a much higher effective tax on innovation and thus enjoying bigger gains from the same change in policy.

3 Quantitative Analysis

In this section I calibrate the model to match several moments from U.S. manufacturing data. A quantitative analysis is useful for two reasons. First, to assess the quantitative effect of patent protection on growth in a model economy that behaves similarly to a real-world economy. Second, to determine whether welfare is increasing or decreasing with patent protection in the U.S.

An important parameter value for this analysis is the royalty rate $\rho$ associated with the strength of patent protection in the U.S. Over the last several decades, there has emerged a rule of thumb used by both license-negotiating firms and courts determining damages owed for infringement, such that a ‘reasonable royalty rate’ is about 25% of the operating profits of the infringer.\(^{12}\) For my calibration I assume the cost to sequential innovators of working around a patent is such that $\rho$ is equal to 25%.

\(^{12}\)See Goldscheider, Jarosz, and Mulhern (2002) for a description of this ‘25 Per Cent Rule’ and a brief history of its emergence.
For the purposes of this analysis, I assume sequential innovators take their quality draws from a one-parameter Kumaraswamy distribution, bounded by 1 and 1 + x, where x continues to be chosen by the firm. This implies a firm-i’s random variable $h_i$ has the following cumulative distribution function:

$$F_i(h) = \text{prob}(h_i < h \mid x_i) = \left(\frac{h - 1}{x_i}\right)^\kappa,$$

where $\kappa$ is a shape parameter for the distribution. In Appendix A.2 I derive the expected value of the best of $N$ draws $E(h_{[1]})$, as a function of $N$ and the level of research per innovation $x$;

$$E(h_{[1]}) = 1 + \frac{\kappa Nx}{\kappa N + 1},$$

and show $\text{prob}(h_i = h_{[1]}) \cdot E(h_{[1]} \mid h_i = h_{[1]})$ can be expressed as;

$$\text{prob}(h_i = h_{[1]}) \cdot E(h_{[1]} \mid h_i = h_{[1]}) = \frac{x_i^{\kappa(N-1)}}{x_{\kappa(N-1)}} \left(\frac{1 + \kappa N(1 + x_i)}{N(\kappa N + 1)}\right).$$

With these distributions in hand, a quantitative analysis requires values for seven exogenous parameters; $\beta$, $\alpha$, $\theta$, $\delta$, $c^S_F$, $c^F_F$, and $\kappa$. Since $N$ is defined as the number of sequential innovations per existing product market, I can normalize the measure of product markets $M$ to 1. To obtain values for the seven required parameters, I therefore need to target six moments from U.S. manufacturing data while ensuring all three equilibrium conditions are satisfied. The first target I use is a real interest rate $r$ of 5%. Given some growth rate $g$, this will determine the value of $\beta = \frac{1+g}{1+r}$. In the model, the share of output retained by firms (and not paid to the factors of production) is $1 - \alpha$. I set $\alpha$ to 0.85, a value commonly used in the growth and development literature. The remaining four targets are a growth rate $g$ of 2.2%, an R&D intensity (research share of output) of 3.1%, a failure rate of newly

\[\text{The Kumaraswamy distribution is similar to the beta distribution, but with simple closed-form density and distribution functions. See Kumaraswamy (1980) and Nadarajah (2008) for details.}\]

\[\text{For example, see Restuccia and Rogerson (2008).}\]
introduced products of 80%, and an elasticity of expected output with respect to R&D of 0.08. The growth target is the average annual growth rate of total factor productivity for U.S. manufacturing from 1977 to 2007, taken from O’Mahony and Timmer (2009), and R&D data is for U.S. manufacturing from 1991 to 2008 from the National Science Foundation. The failure rate of new products is from Daft (2004). Hall, Mairesse, and Mohnen (2010) survey a number of empirical studies of the elasticity of output with respect to R&D and suggest 0.08 as a plausible value. Estimates vary quite a lot, however, so below I discuss the robustness of my results with respect to a range of targets for the elasticity of output with respect to R&D.

In the model the growth rate is $g = E(h_{[1]}) - 1$, given above. Since new-product creators have no research decision, I assume $c_F^S$ accounts for the fixed cost of introducing a product to the market (as in existing markets) and the residual $c_F^0 - c_F^S$ is the fixed cost of research for new-product creators. R&D intensity is therefore equal to $(1 - \delta)N \left( \frac{x^\theta}{\theta} \right) + \delta(c_F^0 - c_F^S)$, multiplied by the measure of product markets $M$. In each period one product is introduced in each new market and $N$ introduced in existing markets. Only one product survives in each market, so the overall failure rate is equal to $1 - \frac{1}{\delta + (1 - \delta)N}$. To derive the elasticity of expected output with respect to research expenditure, note that the equilibrium output of the best firm in a market is a linear function of the firm’s quality draw $h_{[1]}$. The elasticity is therefore;

$$\frac{\partial}{\partial x_i} \text{Prob}(h_i = h_{[1]}) E(h_{[1]}|\cdot) \bigg|_{x_i = x_{-i} = x} \cdot \frac{x}{E(h_{[1]})/N}.$$

The parameter values obtained through the calibration are reported in Table I.

Due to changes in sample design, comparable R&D data is unavailable before 1991.
Table I  Calibrated Parameter Values

<table>
<thead>
<tr>
<th>parameter</th>
<th>description</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>time preference</td>
<td>0.97</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>profit share of output</td>
<td>0.85</td>
</tr>
<tr>
<td>$\theta$</td>
<td>elast. of upper bound w.r.t. R&amp;D</td>
<td>3.42</td>
</tr>
<tr>
<td>$\delta$</td>
<td>prob. of market destruction</td>
<td>0.02</td>
</tr>
<tr>
<td>$c^S_F$</td>
<td>fixed cost of sequential innovation</td>
<td>0.02</td>
</tr>
<tr>
<td>$c^0_F$</td>
<td>fixed cost of new-product creation</td>
<td>0.19</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>distribution parameter</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Using the above parameter values, I can now solve for the competitive equilibrium in an identical economy without patent protection ($\rho = 0$). Table II reports the values of several variables of interest for the economy with and without patent protection. Removing patent protection from the benchmark economy reduces the measure of product markets by 21% and the consumption share of output by a sliver, but substantially increases the growth rate from 2.2% to 2.7%. The net result on welfare is significantly positive. The increase in steady-state utility from removing patent protection is equivalent to the increase in utility consumers would gain from a permanent increase in consumption of 16%.

That the optimal growth rate in an economy depends on the consumer’s discount factor while the equilibrium growth rate does not, implies the welfare predictions of the model are sensitive to the interest rate target, which determines $\beta$. A low interest rate implies a high $\beta$, which in turn implies a high optimal growth rate. With a high optimal growth rate the model is more likely to predict large welfare gains from reduced patent protection. Calibrating the model with an interest rate of 10% (rather than 5%) confirms this, generating an increase in welfare from the elimination of patent protection equivalent to that from an increase in
annual consumption of only 2.7\%\textsuperscript{16}.

For robustness, I also calibrate the model using a wide range of target values for the elasticity of output with respect to R&D. If a target of 0.01 is used, eliminating patent rights causes a larger drop in research intensity while barely changing the results with respect to the change in growth or the measure of product markets. As a result, the estimated increase in welfare is higher. If a target of 0.2 is used, eliminating patent rights causes an increase in research intensity and a slightly lower increase in growth. Welfare still increases, but by slightly less than in the benchmark calibration.

<table>
<thead>
<tr>
<th>Table II Quantitative Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>with patent protection</td>
</tr>
<tr>
<td>--------------------------</td>
</tr>
<tr>
<td>product markets</td>
</tr>
<tr>
<td>number of sequential innovations</td>
</tr>
<tr>
<td>level of research per innovation</td>
</tr>
<tr>
<td>growth rate</td>
</tr>
<tr>
<td>research intensity</td>
</tr>
<tr>
<td>investment rate</td>
</tr>
<tr>
<td>utility</td>
</tr>
</tbody>
</table>

4 Patent Protection, Citations, and Growth

The model developed in Section 2 generates two unambiguous and testable implications. First, an increase in the strength of patent protection should lead to a decrease in productivity growth. Second, greater protection should lead to a decrease in the average quality

\textsuperscript{16}The welfare effect of eliminating patent protection turns negative if the model is calibrated to match an interest rate higher than 15\%.
of innovations. In this section I provide evidence in support of both of these implications. To test whether patent protection is associated with lower productivity growth, I use both a direct measure of growth in industry-level total factor productivity (TFP) and the number of citation-weighted patents generated by a country-industry. To proxy the average quality of innovations I consider the average number of citations per patent.\textsuperscript{17}

4.1 Data

The explanatory variable of interest in this analysis is the strength of patent protection across countries and time. As a measure of patent protection I use the Ginarte-Park Patent Rights Index developed in Ginarte and Park (1997) and updated in Park (2008).\textsuperscript{18} Each country-year in the Index is given a continuous value from zero to five, with the index value increasing in coverage (the number of product-types protected), duration of protection, the scope of patent rights, and membership in international intellectual property treaties, while decreasing in the number of restrictions on patent rights (like compulsory licensing). The Index reports values for each country every five years from 1960 to 2005.

TFP data is from O’Mahony and Timmer (2009), and contains measures of TFP growth for 32 industry groups across 15 countries from 1970 to 2007.\textsuperscript{19} Given the five-year intervals in the Patent Index, I use an average of the growth rates reported between the years used in the Index (for example, 1980 to 1984).\textsuperscript{20} The index value I use for each interval in the analysis is an average of the index values bounding the interval (1980 and 1985). For each country-industry-‘year’ observation I also construct a control variable containing an average

\textsuperscript{17}See Hall, Jaffe, and Trajtenberg (2005) for a discussion and defense of the use of patent citations as a proxy for patent quality.

\textsuperscript{18}The Ginarte-Park Index has been used widely. Some examples are Smith (2001), Branstetter, Fisman, and Foley (2006), and Antràs, Desai, and Foley (2009).

\textsuperscript{19}O’Mahony and Timmer (2009) provide TFP data for 18 countries. The fifteen countries used here include Australia, Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Japan, the Netherlands, Spain, Sweden, the United Kingdom, and the United States. The Czech Republic, Hungary, and Slovenia are not used due to a lack of patent protection data.

\textsuperscript{20}Observations after 2004 are not used.
of TFP growth rates across all other countries for the industry-‘year’ in question.\textsuperscript{21} The resulting unbalanced panel of country-industry-year observations contains six time periods. Table III reports descriptive statistics for this sample.

### Table III Descriptive Statistics: TFP Data

<table>
<thead>
<tr>
<th></th>
<th>10th</th>
<th>90th</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>s.d.</td>
</tr>
<tr>
<td>Patent Index</td>
<td>3.9</td>
<td>0.6</td>
</tr>
<tr>
<td>TFP growth rate</td>
<td>1.3</td>
<td>5.0</td>
</tr>
<tr>
<td>time periods</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>countries</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>industries</td>
<td>32</td>
<td></td>
</tr>
</tbody>
</table>

All patent and citation data is from the NBER Patent Database, which contains observations for each patent applied for in the U.S. from 1976 to 2006.\textsuperscript{22} Each observation contains information about the origin country (or countries) of each patent, the application year, the number of citations received by the patent in subsequent years, and the unique 3-digit Standard Industrial Classification code associated with the patent (I throw out observations missing any of this information). The analyses here will also use country-industry-year observations. As with the TFP data, I pool observations across the five years between the years used in the Index.\textsuperscript{23} For each observation of each independent variable of interest $y_i$ (where $y$ could be citation-weighted patent counts or citations per patent) I also construct a control variable $y_{not-i}$ using all other patents not used to construct $y_i$.\textsuperscript{24} I do not include the

\textsuperscript{21}For the remainder of this section I use ‘year’ to refer to a 5-year time period.
\textsuperscript{22}See Hall, Jaffe, and Trajtenberg (2005) for further details.
\textsuperscript{23}For the first time interval, I use 1976 to 1979 and simply scale up the number of citations and patents by 5/4.
\textsuperscript{24}For citation-weighted patent counts, the control variable contains total counts across all other countries. For citations per patent, the control variable contains total citations divided by total patents across all other countries.
U.S. in this analysis because the level of patent protection in the U.S. presumably affects the behavior of foreign firms patenting in the U.S., which means an appropriate control variable $y_{not-US}$ cannot be constructed. Finally, I drop all patents with multiple countries of origin.

<table>
<thead>
<tr>
<th>Table IV Descriptive Statistics: Patent Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Patent Index</td>
</tr>
<tr>
<td>citation-weighted patents</td>
</tr>
<tr>
<td>citations per patent</td>
</tr>
<tr>
<td>time periods</td>
</tr>
<tr>
<td>countries</td>
</tr>
<tr>
<td>industries</td>
</tr>
</tbody>
</table>

Once each of the above adjustments have been made, I am left with an unbalanced panel of country-industry-year observations made up of 87 countries and 452 industries over 6 time periods. Table IV provides descriptive statistics for the sample. The average index value across country-years is 3.0, with a great deal of variation across countries and over time. The average number of citation-weighted patents across country-industry-years is 38, with a distribution skewed to the right. I treat observations with zero patents in the following way. Until a country-industry reports a positive number of patents, I assume no activity in that industry for that country and treat the number of patents as missing. Once a country-industry reports a positive number of patents, I treat any future zero observations for that country-industry as zeroes. Interpreted in this way, almost one quarter of all country-industry-years report zero citation-weighted patents.
4.2 Econometric Models

4.2.1 Productivity Growth

To test the effect of patent protection on the growth rate of industry-level TFP, I estimate the following equation;

$$y_{c,i,t} = \sum \alpha_c \delta_c + \beta_1 \overline{y}_{i,t} + \beta_2 \text{index}_{c,t} + u_{c,i,t},$$

where $y_{c,i,t}$ is TFP growth for industry $i$ in country $c$ at time $t$, $\delta_c$ is a dummy variable for country $c$, and $\overline{y}_{i,t}$ is the control variable described above. The measure of patent protection used here is constant across industries within a given country in a given time period, but using separate observations of TFP growth for each industry is still useful for two reasons. First, this controls for variation in the composition of industries across countries. Second, this allows for the inclusion of the control variable $\overline{y}_{i,t}$ which controls for exogenous differences in growth rates across industries and over time. The inclusion of a country dummy controls for differences in growth rates across countries due to factors other than patent protection. Endogeneity is still a potential problem for two reasons. First, the political decision to strengthen patent rights could very well be influenced by the level of innovation and growth (in particular, firms may lobby for protection when expenditure on innovation is high). Second, the Ginarte and Park Patent Rights Index allocates a higher index value to countries ratifying international property rights treaties. Ratification of these treaties is often accompanied by, and even a condition for, entry into the World Trade Organization or other trade agreements which could themselves increase the incentive for firms to innovate. Not accounting for these problems should lead to upwardly-biased estimates of the effects of patent protection. To address this potential endogeneity I instrument the index value for each country-year with the previous year’s value, and report both OLS and 2SLS estimates of the above equation.
4.2.2 Patents

As is standard in the patent-count literature, I assume citation-weighted patents follow a Poisson process with the following hazard rate;

\[ \exp \left\{ \sum \alpha^C \delta^C_t + \sum \alpha^I \delta^I_t + \beta_1 y_{i,t} + \beta_2 \text{index}_{i,t} \right\} , \]

where variables are defined as above. The control variable here helps to control for exogenous changes in within-industry growth rates over time, but does little to control for differences in patenting activity across industries. To control for these differences I therefore include an industry dummy variable \( \delta^I \). To address the endogeneity discussed above I instrument the index value for each country-year with the previous year’s value, using a control function approach as in Aghion et al. (2005).

For my test of the effect of patent protection on citations per patent, I use a weighted least squares estimator with a country dummy and the control variable described above, and with observations weighted by the number of patents used to construct the observation. Faced with the same endogeneity issues I also report estimates from a two stage weighted least squares estimator, using the same instrument described above.

4.3 Results

The first two columns of Table V report the results of the OLS and 2SLS regressions for industry-level TFP growth. The estimated coefficients for patent protection are both statistically and economically significant, and are consistent with the conclusions of the model developed in Section 2. Given that O’Mahony and Timmer (2009) also report measures of aggregate TFP for the fifteen countries used here, I also report the results of both OLS and 2SLS regressions using aggregate growth rates in the last two columns of Table V.\(^\text{25}\) Although

\(^{25}\)Country dummies are included in these regressions, but the control variable has been omitted.
the standard errors are higher (understandable, given the large reduction in observations),
the estimated coefficients on patent protection are similar.

<table>
<thead>
<tr>
<th>Table V Results: TFP Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>dependent variable:</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>instrument used:</td>
</tr>
<tr>
<td>Patent Index</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>control variable</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$R^2$ (adjusted)</td>
</tr>
<tr>
<td>observations</td>
</tr>
</tbody>
</table>

All regressions include country dummies, but dummy coefficients are omitted. Robust standard errors reported in parentheses.

***, **, and * denote significance at 1%, 5%, and 10% levels.

Table VI reports the results of two regressions using citation-weighted patents, with and without controlling for endogeneity. The estimated coefficients for patent protection suggest that quality-adjusted patents decrease when patent protection increases. In particular, the estimated coefficient in the second regression suggests an increase in patent protection by one standard deviation from an average value of 3 is associated with a drop in citation-weighted patents from 38 to 23. The effect of patent protection on citation-weighted patents is both statistically significant and large in magnitude, consistent with both the model in Section 2 and the results reported in Table V.
### Table VI Results: Patents

<table>
<thead>
<tr>
<th>dependent variable:</th>
<th>citation-weighted patents</th>
<th>instrument used:</th>
<th>no</th>
<th>yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patent Index</td>
<td>-0.38***</td>
<td>-0.54***</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>control variable</td>
<td>0.0005***</td>
<td>0.0005***</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
</tbody>
</table>

| significance of Index | 34934 | (0.00) |
| significance of instrument | 111000 | (0.00) |
| in reduced form      |       |       |
| control function     | 0.62  | (0.01) |
| $R^2$                | 0.72  | 0.73 |
| $R^2$ of reduced form|       | 0.88 |
| observations         | 37517 | 36349 |

Both regressions include country and industry dummies, but dummy coefficients are omitted. Estimates from Poisson regressions, with standard errors in parentheses. Robust standard errors reported in first column, but errors in second column have not been corrected for inclusion of control function. Significance tests show likelihood ratio test-statistics with $P$-values in parentheses.

*** denotes significance at 1% level.

Table VII reports the results of two regressions for the number of citations per patent. The estimated coefficients for patent protection are negative and statistically significant.
The estimate in the last column suggests an increase in patent protection by one standard deviation from an average value of 3 is associated with a drop in the number of citations per patent from an average of 5.2 to an average of 3.9. This is consistent with the model in Section 2, where stronger patent protection leads to a drop in the average quality of innovations.

Table VII Results: Citations per Patent

<table>
<thead>
<tr>
<th>dependent variable: citations per patent</th>
<th>instrument used: no</th>
<th>yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patent Index</td>
<td>−1.13***</td>
<td>−1.43***</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>control variable</td>
<td>0.83***</td>
<td>0.81***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>$R^2$ (adjusted)</td>
<td>0.54</td>
<td>0.54</td>
</tr>
<tr>
<td>observations</td>
<td>32111</td>
<td>31186</td>
</tr>
</tbody>
</table>

Both regressions include country dummies, but dummy coefficients are omitted. Robust standard errors reported in parentheses. *** denotes significance at 1% level.

5 Conclusion

In this paper I pull together ideas from the theoretical literatures on competition, innovation, growth, and patent rights to develop a comprehensive yet tractable model consistent with key facts documented in recent empirical studies. Given the tendency for cross-country intellectual property treaties to be tied to trade agreements and market reforms, a comprehensive framework of the kind developed here is necessary to disentangle the potentially contradictory effects of the resulting changes in multiple policies. In the model, as in the
data, an increase in competition driven by product-market deregulation is associated with more innovation and higher growth. An increase in the strength of patent rights results in less sequential innovation and lower growth while expanding the equilibrium variety of products. While the model is ambiguous with respect to the effect of patent rights on welfare, calibrating the model generates the prediction that eliminating patent rights should result in a substantial increase in welfare. The model generates an additional testable implication, that stronger patent protection should lead to a lower average quality of innovations. I offer evidence that this is indeed the case.

The theoretical and empirical results reported here support Boldrin and Levine’s (2008b) recent claim that evidence of a positive link between competition and innovation is itself evidence against the need for patent protection to encourage innovation. One must wonder at the foresight of a commenter who reached similar conclusions two centuries ago;

In some other countries [the granting of patent rights] is sometimes done, in a great case, and by a special and personal act, but, generally speaking, other nations have thought that these monopolies produce more embarrassment than advantage to society; and it may be observed that the nations which refuse monopolies of invention, are as fruitful as England in new and useful devices.

- Thomas Jefferson, letter to Isaac McPherson, 13 August 1813

WEST VIRGINIA UNIVERSITY
A Appendix

A.1 Expected Value of the Best Draw: General

In existing markets a sequential innovator-\(i\) receives a quality \(A_i\) equal to \(A_{[i],-1} \cdot h_i\), where \(A_{[i],-1}\) is the best quality of the previous period and \(h_i\) is the realization of a random variable drawn from a distribution with support 1 and \(1 + x_i\). Let \(f_i(h)\) and \(F_i(h)\) denote the pdf and cdf of an innovator-\(i\)’s draw, conditional on \(x_i\). It will be useful to transform these functions in the following way. Define a new random variable \(\hat{h} \in (0, 1)\) such that \(h_i = 1 + x_i \cdot \hat{h}_i\), so \(\hat{f}(\hat{h})\) and \(\hat{F}(\hat{h})\) are independent of firm-\(i\)’s level of research \(x_i\). Each firm chooses the same level of research \(x\) in equilibrium, so deriving the expected value of the best draw \(h_{[1]}\) conditional on \(x\) and the number of draws \(N\) requires only a density function for \(\hat{h}_{[1]}\):

\[
f(\hat{h}_{[1]} = v_1 \mid x, N) = \int_0^{v_1} \cdots \int_0^{v_{N-1}} N! \prod_{\ell=1}^{N} \hat{f}(\hat{h} = v_\ell) du_N \cdots du_2,
\]

or

\[
f(\hat{h}_{[1]} = v_1 \mid x, N) = N \hat{f}(v_1) \hat{F}(v_1)^{N-1},
\]

defined over \(v_1 \in (0, 1)\). The expected value of the best draw is therefore;

\[
E(h_{[1]}) = 1 + x \cdot N \int_0^{1} v \hat{f}(v_1) \hat{F}(v_1)^{N-1} dv_1.
\]

The above expression is obviously increasing in the level of research per innovation \(x\).

A.2 Expected Value of the Best Draw: Kumaraswamy

For the purposes of the calibration in Section 3, I assume sequential innovators take their quality draws from a one-parameter Kumaraswamy distribution, bounded by 1 and \(1 + x\), where \(x\) is chosen by the firm. Each firm-\(i\)’s random variable \(h_i\) therefore has the following
cumulative distribution function;

\[ F_i(h) = \text{prob}(h_i < h \mid x_i) = \left( \frac{h - 1}{x_i} \right)^\kappa, \]

and the following probability density function;

\[ f_i(h) = \text{prob}(h_i = h \mid x_i) = \frac{\kappa(h - 1)^{\kappa - 1}}{x_i^\kappa}, \]

where \( \kappa \) is a shape parameter for the distribution.

To solve the model, it is necessary to derive the joint density function of \( h_i \) and \( h_{[1]} \);

\[
f(h_{[1]} = v_1, h_1 = h_{[1]} \mid x_i, x_{-i}, N) = \int_1^{v_1} \cdots \int_1^{v_{N-1}} (N - 1)! f(h_i = v_1) \prod_{\ell=2}^N f(h_{-i} = v_\ell) dv_N \cdots dv_2,
\]

where \( N \) is the number of firms (draws), \( x_i \) is firm-\( i \)'s level of research, and \( x_{-i} \) is every other firm’s level of research. Integrating over all \( v_{\ell>1} \) and using the density function given above, this simplifies to;

\[
f(h_{[1]} = v_1, h_1 = h_{[1]} \mid x_i, x_{-i}, N) = \frac{\kappa(v_1 - 1)^{\kappa N - 1}}{x_i^\kappa x_{-i}^{\kappa(N-1)}}.
\]

The density function of \( h_{[1]} \) is derived in a similar way, but without the qualification that firm-\( i \) has the highest draw;

\[
f(h_{[1]} = v_1 \mid x, N) = \int_1^{v_1} \cdots \int_1^{v_{N-1}} N! \prod_{\ell=1}^N f(h = v_\ell) dv_N \cdots dv_2,
\]

or

\[
f(h_{[1]} = v_1 \mid x, N) = \frac{\kappa N(v_1 - 1)^{\kappa N - 1}}{x^\kappa N}.
\]
The density functions above can now be used to derive the following:

\[
\text{prob}(h_i = h_{[1]}) \cdot E(h_{[1]} | h_i = h_{[1]}) = \frac{x_i^{\kappa(N-1)}}{x_{-i}^{\kappa(N-1)}} \left( \frac{1 + \kappa N(1 + x_i)}{N(\kappa N + 1)} \right),
\]

and

\[
E(h_{[1]}) = 1 + \frac{\kappa N x}{\kappa N + 1}.
\]

### A.3 Constrained Social Planner’s Problem

Here I solve a constrained social planner’s problem to characterize the optimal investment rate and optimal amount of investment in sequential innovation, where the social planner is bound by the constraint that the relationship between the number of innovations per market \(N\) and the level of research per innovation \(x\) is identical to that in the decentralized economy. Given my focus on the stationary equilibrium of the economy, the constrained social planner’s problem is to choose the investment rate \(I\) and the per-market level of investment in sequential innovation \(X\) to maximize:

\[
U = \sum_{t=0}^{\infty} \beta^t u(C_t) = \sum_{t=0}^{\infty} \beta^t \log \left[ M^{\frac{1-\alpha}{\alpha}} \cdot (1 + g)^t \cdot (1 - I) \right],
\]

or

\[
U = \frac{\log(1 - I)}{1 - \beta} + \frac{(1 - \alpha)}{\alpha(1 - \beta)} \log(M) + \frac{\beta}{(1 - \beta)^2} \left( \frac{1 - \alpha}{\alpha} \right) \log(E(h_{[1]})),
\]

s.t. \(M = \frac{I}{\delta c_F^0 + (1 - \delta)X}\),

where \(X = N \left( c_S^F + \frac{x^S}{\theta} \right)\).

The first-order conditions for the above problem can be manipulated to find that the optimal investment rate \(I\) is equal to \(1 - \alpha\). It is immediately clear that the investment rate in the decentralized economy is lower without patent protection, and even lower with patent.

\(^{26}\)I normalize \(A_0\) to 1 for all markets in period 0.
protection. Optimal investment in sequential innovation per market $X$ is characterized by the following equation;

$$\frac{\partial}{\partial X} \frac{E(h_{[1]})}{E(h_{[1]})} = \frac{(1 - \delta)(1 - \beta)}{\beta(1 - \alpha)}.$$
References


