Competition as a Discovery Procedure:
Schumpeter Meets Hayek in a Model of Innovation

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Abstract

I incorporate an insight of Friedrich Hayek — that competition allows a thousand flowers to bloom, and discovers the best among them — into a model of Schumpeterian innovation. Firms face uncertainty about the optimal direction of innovation, so more innovations implies a higher expected value of the ‘best’ innovation. The model accounts for two seemingly contradictory relationships reported in recent empirical studies — a positive relationship between competition and industry-level productivity growth, and an inverted-U relationship between competition and firm-level innovation. Notwithstanding the positive relationship between competition and growth, I find antitrust policy reduces industry-level growth.

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1 Introduction

Schumpeter (1942) argues that the expectation of monopoly power is necessary to induce innovation. This suggests a higher level of competitive rivalry should translate into lower levels of innovation, and thus lower productivity growth. Workhorse models of innovation all feature this Schumpeterian mechanism and share the conclusion that competition is harmful to growth.\(^1\) Recent empirical studies, however, come to very different conclusions. Productivity growth at the industry level has been shown to be positively correlated with competition, while the average level of innovation per firm in an industry exhibits an inverted-U relationship with competition — that is, a positive relationship when competition is relatively low, and a negative relationship when competition is high.\(^2\) The objective of this paper is to develop a model of innovation that is consistent with these two stylized facts.

There are two key features of the model developed here relative to the existing literature. First, innovators are uncertain about the relative value of each possible direction of innovation, until an innovation has actually been introduced to the market. In such an economy, more firm entry induced by lower entry barriers results in a greater number of innovations tried, which in turn implies a higher expected value of the ‘best’ innovation. If the best innovation can capture a market through Bertrand competition (the second key feature of the model), it becomes possible that measured productivity growth for an industry can be increasing in the number of firms, even if the average level of research per firm is declining (due to the usual Schumpeterian mechanism). These features capture the insight of Hayek (2002) that competition allows a thousand flowers to bloom, and discovers the best among

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\(^1\)For example Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). More recent models of R&D-based endogenous growth like Dinopoulous and Thompson (1998), Connolly and Peretto (2007), and Etro (2007) continue to share the implication that lower entry barriers (and thus more competition) should lower the growth rate.

Key here is the distinction between uncertainty about how to accomplish a given goal (i.e., how to improve a product along certain dimensions), and uncertainty about the relative values of competing goals (i.e., which dimensions to improve upon). While experimenting with different ways of achieving a goal may lead to a better outcome, such experimentation can be done within a firm when outcomes are easily measurable. On the other hand, if the relative values of competing innovations can only be determined after they are introduced to the market, firms face a strong disincentive to experiment with competing products. As a result, experimentation will arise primarily through the introduction of new products or production processes by multiple firms. In such a setting, competition works to ‘discover’ the best innovation.

In the model economy, innovators in each product market introduce improved variations of an existing product in an attempt to capture the market. Innovators invest in research to increase the magnitude of their improvements, but remain uncertain about which new variant will capture the market until the cost of entry is incurred. The expected value of the best innovation depends on both the level of research per firm and the number of firms introducing innovations. Motivated by Aghion et al. (2005) who test the effects on firm-level innovation from stronger competition driven by product-market deregulation, I consider the effects on innovation in the model from a decrease in entry barriers to new innovators. I show that lower entry barriers increase the number of innovations, increase industry-level productivity growth, lower markups (the measure of competition in Aghion et al.), and first increase but then decrease firm-level innovation. I further extend the model to allow for

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3In Hayek’s words, competition is “a procedure for discovering facts which, if the procedure did not exist, would remain unknown or at least would not be used.” (2002, p.9)

4The intuition here is straightforward. Consider a firm that is introducing one product and has some expectation of a resulting stream of profits. If this same firm was to introduce a second product in direct competition, it would reduce the expected profits from the first product. Such a firm would be better off introducing a differentiated second product, not in direct competition with the first.

5Daft (2004) reports 80 percent of new products fail in their first year, suggesting this uncertainty plays an important role in the competitive process.
collusion between firms and find that antitrust policy making collusion more costly reduces growth, notwithstanding the positive effects of lower entry barriers to innovators. Although consistent with conventional wisdom, this conclusion is in stark contrast to those made in many empirical studies of competition and growth.

The model developed in this paper is closest to Grossman and Helpman (1991). While Grossman and Helpman incorporate uncertainty in the timing of innovations, keeping the magnitude of innovations constant, I keep the timing constant but make the magnitude of each innovation uncertain. This allows for an endogenously determined number of innovators per market, while in Grossman and Helpman the number of innovators is indeterminate. Aghion et al. (2005) develop a duopoly model which generates an inverted-U between competition and firm-level innovation. The present paper incorporates endogenous entry, while Etro (2007) suggests the inverted-U in Aghion et al. disappears once entry is allowed for. To my knowledge, no paper that allows for the free entry of firms has found a mechanism through which lower entry barriers could result in higher growth either at the firm level or at the level of an industry. Boldrin and Levine (2002, 2005, 2008) study the conditions under which innovation can occur under perfect competition, while the present paper considers variation in the level of competition.

In the next section I describe the model and characterize its equilibrium. In Sections 3 and 4 I discuss the equilibrium relationships between competition, innovation, and productivity growth, and discuss the empirical evidence that supports the central mechanism in the model and its implications. I extend the model in Section 5 to evaluate the effects of antitrust policies on growth. Section 6 concludes.

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Peretto (1996) can generate an increase in growth and in the number of firms when fixed operating costs are reduced, but requires an assumption that a firm’s R&D directly improves the productivity of rival firms. Peretto (1999) shows this result disappears when spillovers are removed from the model.
2 The Model

Consider an economy in which a final good is produced using a continuum of inputs (of measure one) from a representative intermediate industry. Intermediate firms (hereafter referred to as ‘firms’) produce these inputs one-for-one using labor. The final good can be used for consumption or innovation, and will also act as the numéraire. There are a large number of potential innovating firms, any of which can choose to introduce an improved version of an existing product. I study the equilibrium of the economy along a balanced growth path, in which firms take the economy-wide growth rate and interest rate as given, and free entry ensures zero expected profits for all entrants. I begin by describing the environment.

2.1 Environment

There is a representative consumer who inelastically supplies one unit of labor to each product market. The consumer only values consumption $C$ and has a constant discount factor $\beta \in (0, 1)$. Preferences over the stream of consumption in each period are described by the following log-utility function:

$$\sum_{t=0}^{\infty} \beta^t \log(C_t).$$

Product markets are indexed by $m$. At any point in time $t$ there are $N_{m,t}$ firms able to produce good-$m$, but the output of each firm-$i$ in market $m$, $i \in \{1, ..., N_m\}$, is associated with a firm-specific quality denoted by $A_{i,m}$. I assume variants of good-$m$ are perfectly substitutable so only the firm with the 1st-best quality $A_{[1],m}$ will produce in equilibrium.

The market for final output is perfectly competitive, with a representative firm using inputs from a representative intermediate industry to produce output according to the fol-
lowing production function;

\[ Y_t = \left( \int_0^1 (A_{[1],m,t} y_{m,t})^\alpha dm \right)^{1/\alpha}, \]

where \( y_m \) is the quantity of input-\( m \) used and \( \frac{1}{1-\alpha} \) is the constant elasticity of substitution between differentiated product markets.

### 2.2 Innovation

In each period, after both intermediate inputs and the final good have been produced, any firm can choose to undertake the investment necessary to create and introduce an improved product in the subsequent period. Any firm-\( i \) wishing to incur the cost of introducing an improved version of product-\( m \) in period \( t \) receives a quality equal to \( A_{i,m,t} = A_{[1],m,t-1} \cdot h_{i,m,t} \), where \( h_{i,m,t} \) is a random variable distributed uniformly between 0 and \( x_{i,m,t-1}^\theta \), \( x \) is chosen by the firm, and \( \theta \in (0,1) \) is the elasticity of the upper bound with respect to research expenditure.\(^8\) I assume the cost of research is \( \Psi_{m,t-1} \cdot c_X \cdot x_{i,m,t-1} \).\(^9\) Before realizing the value of its draw (as well as the draws of other firms) an innovating firm must also incur a fixed cost of introducing its product to market \( m \) equal to \( \Psi_{m,t-1} \cdot c_F \).\(^{10}\) Only once all of these costs are sunk do firms learn their own and each other’s quality. By multiplying these costs by \( \Psi_{m,t-1} \equiv Y_{t-1} \cdot \frac{A_{0}^\alpha_{[1],m,t-1}}{\int_0^1 A_{[1],m',t-1}^\alpha dm'} \), I ensure innovating firms will face the same decision each period.

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\(^8\)Changing the support of \( h \) to \( (1, 1+x^\theta) \) complicates the analysis without changing any of the qualitative results. Allowing innovators with very low draws to use \( A_{[1],t-1} \) does the same.

\(^9\)Throughout the paper, I will often refer to a firm’s choice of \( x \) as that firm’s ‘level of research’.

\(^{10}\)Introducing a new product to the market might entail holding an inventory for a time before demand is realized, or fulfilling regulatory requirements like efficacy trials for drugs.
2.3 Equilibrium

I focus on the equilibrium of the economy along a balanced growth path. In such an equilibrium the interest rate \( r \) and growth rate \( g \) are constant, and the average wage \( w \) grows at the same rate as total output. In addition, the assumptions made about the environment above will ensure both a constant number of sequential innovators \( N \) per market in each period and a constant level of research per innovation \( x \).\(^{11}\) I begin by describing the decision problems of each agent, and then define and solve for equilibrium.

2.3.1 Consumer

In each period the consumer chooses both consumption and savings, and the only vehicle for savings is the purchase of equity in innovating firms, earning a rate of return of \( r \).\(^{12}\) The consumer’s problem is therefore to choose consumption \( C \) and savings \( S \) in each period \( t' \), given \( w, r, \) and \( g \), to maximize;

\[
\sum_{t=t'}^{\infty} \beta^t \log(C_t), \quad \text{s.t.} \quad C_t + S_t \leq w_{t'}(1 + g)^{t-t'} + S_{t-1}(1 + r).
\]

The first order conditions for this problem imply the following interest rate;

\[
r = \frac{1 + g}{\beta} - 1.
\]

2.3.2 Final-Good Producer

In each period, the final-good producer takes the prices of all intermediate inputs as given, and demands inputs from each intermediate firm to maximize profits;

\[
Y - \int_0^1 P_m y_m dm,
\]

\(^{11}\)I henceforth omit the time subscript unless clarity requires it.
\(^{12}\)I take it as given that the consumer will diversify across all innovating firms, as they will all share the same expected value before innovating.
where \( Y \equiv \left( \int_0^1 (A_{[1],m} y_m)^{\alpha} \, dm \right)^{\frac{1}{\alpha}} \) and \( P_m \) is the price of input-\( m \). The first order conditions for the final-good firm’s problem imply the following inverted demand function for each intermediate input:\(^{13}\)

\[
P_m = Y^{1-\alpha} A_{[1],m}^{\alpha} y_m^{\alpha-1}.
\]

### 2.3.3 Intermediate Firms

In each period, once the quality of each innovator is realized, the highest-quality firm in each product market can capture its market by choosing a price less than or equal to each rival’s quality-adjusted marginal cost. This ‘best’ firm faces the downward-sloping demand curve implied by the final-good firm’s problem above, and demands labor \( y_m \) given the wage \( w_m \), to maximize operating profits;

\[
\pi_m = P_m y_m - w_m y_m, \quad \text{s.t.} \quad P_m \leq w_m \frac{A_{[1],m}}{A_{[2],m}},
\]

where \( A_{[\ell],m} \) denotes the \( \ell \)-th best quality in market \( m \). The firm’s optimal price therefore depends on the ratio \( A_{[1],m}/A_{[2],m} \) in the following way;

\[
P_m = w_m \cdot \min \left\{ \frac{1}{\alpha}, \frac{A_{[1],m}}{A_{[2],m}} \right\}.
\]

When the quality of the second-best firm is far enough away from the best, the optimal price is just a fixed markup over marginal cost, as in a standard model of monopolistic competition. When the best and second-best firms are close rivals, the above constraint is binding and the optimal price is equal to the second-best firm’s quality-adjusted marginal cost.

Taking advantage of the assumption that labor (and therefore output) is fixed and equal to one in each product market, the demand curve and optimal price can be used to derive

\(^{13}\)I have taken it for granted here that only one firm will produce in each product market.
the operating profits of the best firm in market \( m \);

\[
\pi_m = \begin{cases} 
A^\alpha_{[1],m}(1-\alpha)Y^{1-\alpha}, & \text{if } \frac{A^2_{[2],m}}{A^1_{[1],m}} < \alpha \\
\left(\frac{A^1_{[1],m} - A^2_{[2],m}}{A^1_{[1],m}}\right)Y^{1-\alpha}, & \text{if } \frac{A^2_{[2],m}}{A^1_{[1],m}} > \alpha
\end{cases}
\]

By taking into account \( A_{i,m,t} = A_{[1],m,t-1} \cdot h_{i,m,t} \), the expected value of investing in innovation in period \( t-1 \) to introduce an improved product into market \( m \) in period \( t \) for some firm-\( i \) can now be expressed as:

\[
V_{i,m,t-1} = -\Psi_{m,t-1} \left( c_F + c_X x^\theta_{i,m,t-1} \right) + \frac{Y^{1-\alpha} A^\alpha_{[1],m,t-1}}{1+r} \text{Prob} \left[ h_{i,m,t} = h_{[1],m,t} > \frac{h^2_{[2],m,t}}{\alpha} \right] \cdot E_{t-1} \left[ (1-\alpha)h^\alpha_{[1],m,t} \right] + \frac{Y^{1-\alpha} A^\alpha_{[1],m,t-1}}{1+r} \text{Prob} \left[ h_{i,m,t} = h_{[1],m,t} < \frac{h^2_{[2],m,t}}{\alpha} \right] \cdot E_{t-1} \left[ \frac{h^1_{[1],m,t} - h^2_{[2],m,t}}{h^{1-\alpha}_{[1],m,t}} \right],
\]

where \( \Psi_{m,t-1} \equiv Y_{t-1} \cdot \frac{A^\alpha_{[1],m,t-1}}{\int_0^1 A^\alpha_{[1],m',t-1} dm'} \). Given a firm-\( i \)'s decision to introduce an improved product, it will choose its level of research \( x_i \) to maximize the above value function, resulting in the following optimal research condition:

\[
\frac{\partial}{\partial x_{i,m,t-1}} V_{i,m,t-1} = 0.
\]

### 2.3.4 Definition of Equilibrium

An equilibrium along a balanced growth path is a constant interest rate \( r \), growth rate \( g \), number of innovators per market \( N \), level of research per innovation \( x \), and output per market \( y_m \), and invariant distributions of the prices \( P_m \) and wages \( w_m \) across markets, such that the following conditions are satisfied:

(i) Consumer Optimization: \( 1 + r = \frac{1+g}{\beta} \)

(ii) Final-Good Firm Optimization: \( P_m = Y^{1-\alpha} A^\alpha_{[1],m}, \forall m \in [0, 1] \)
(iii) Intermediate Producer Optimization: \( \frac{P_m}{w_m} = \min \left\{ \frac{1}{\alpha} A_{[1],m} \right\}, \forall m \in [0,1] \)

(iv) Optimal Research: \( \frac{\partial}{\partial x_{i,m}} V_{i,m} = 0, \forall m \in [0,1] \)

(v) Free Entry: \( V_{i,m} = 0, \forall m \in [0,1] \)

(vi) Market Clearing (Goods): \( Y = \left( \int_0^1 A_{[1],m} dm \right)^{\frac{1}{\alpha}} \)

(vii) Market Clearing (Labor): \( y_m = 1, \forall m \in [0,1] \)

I start by solving for the partial equilibrium of the representative industry for a given interest rate \( r \) and growth rate \( g \), in anticipation of the partial equilibrium analysis in Section 3. From condition (vi) it is clear that;

\[
Y_t^{1-\alpha} A_{[1],m,t-1} = \frac{Y_{t-1}(1+g)^{1-\alpha} A_{[1],m,t-1}}{\left( \int_0^1 A_{[1],m} dm \right)^{\frac{1}{\alpha}}} = \Psi_{m,t-1}(1+g)^{1-\alpha},
\]

so the expected value of introducing an improved product to market \( m \) for some firm-\( i \) is;

\[
V_{i,m,t-1} = -\Psi_{m,t-1} \left( c_F + c_X A_{i,m,t-1} \right)
+ \Psi_{m,t-1}(1+g)^{1-\alpha} \frac{\text{Prob} \left[ h_{i,m,t} = h_{[1],m,t} > \frac{h_{[2],m,t}}{\alpha} \right]}{1+r} \cdot E_{t-1}\left[ (1-\alpha)h_{[1],m,t} \right] \\
+ \Psi_{m,t-1}(1+g)^{1-\alpha} \frac{\text{Prob} \left[ h_{i,m,t} = h_{[1],m,t} < \frac{h_{[2],m,t}}{\alpha} \right]}{1+r} \cdot E_{t-1}\left[ \frac{h_{[1],m,t} - h_{[2],m,t}}{h_{[1],m,t}} \right].
\]

Given the assumed distribution from which firms draw, the (partial) equilibrium number of innovators per market \( N \) and level of research per innovation \( x \) can now be characterized using conditions (iv) and (v) above:\(^{14}\)

\[
\text{Optimal Research (partial eq.): } x = \left[ \frac{(1+g)^{1-\alpha} (1-\alpha^N) \theta (N + \alpha - 1)}{c_x (1+r) N (N + \alpha)} \right]^{\frac{1}{1-\alpha^N}}
\]

\(^{14}\)The derivation of these two equations are explained in more detail in Appendix A.2 after first deriving the joint distribution of the best and second-best quality draws in Appendix A.1.
Free Entry (partial eq.): \( c_F + c_x x = \frac{(1 + g)^{1-\alpha}(1 - \alpha^N)x^{\alpha\theta}}{(1 + r)N(N + \alpha)} \).

All other variables are functions of \( N \) and \( x \).

To solve for the general equilibrium of the economy I first note the growth rate \( g \) is equal to;

\[
g = \frac{Y_t}{Y_{t-1}} - 1 = \left( \frac{\int_0^1 A_{[1],m,t-1}^{\alpha} h_{[1],m,t}^{\alpha} dm}{\int_0^1 A_{[1],m,t}^{\alpha} dm} \right)^{\frac{1}{\alpha}} - 1.
\]

In Appendix A.2 I show this reduces to;

\[
g = E(h_{[1]}^{\alpha})^{\frac{1}{\alpha}} - 1 = \left( \frac{N x^{\alpha\theta}}{N + \alpha} \right)^{\frac{1}{\alpha}} - 1.
\]

Combining this expression for \( g \) with equilibrium conditions (i), (iv), and (v), I can now characterize the general equilibrium number of innovators per market and the level of research per innovation;

Optimal Research (general eq.): \( x = \frac{\beta(1 - \alpha^e)\theta(e + \alpha - 1)}{me^2} \)

Free Entry (general eq.): \( c_F + c_x x = \frac{\beta(1 - \alpha^N)}{N^2} \).

3 Results

In this section I discuss how equilibrium depends on exogenous variables and parameters. The empirical studies motivating this paper rely on within-industry variation over time to identify the relationships they report between competition, innovation, and productivity growth. It is therefore appropriate to focus on the effects of changes in industry-specific parameter values on outcomes in a single industry, and assume this single industry is too small to affect the rest of the economy. Given my decision to use a representative industry in the model, I therefore examine how the partial equilibrium of this industry depends on the
model’s parameters, while keeping fixed the interest rate and growth rate of the economy. I focus on the effect of changes in $c_F$, the fixed cost of introducing an innovation, as the most plausible driver of the relationships identified in the data.\textsuperscript{15} In Appendix A.4 I prove the following results hold for all parameter values (for which equilibria exist); (i) the equilibrium number of firms is decreasing in $c_F$; (ii) the level of competition (measured as in Aghion et al. (2005)) is decreasing in $c_F$; (iii) both research per firm and the average level of innovation per firm exhibit an inverted-U relationship with $c_F$; and (iv) industry-level productivity growth is decreasing in $c_F$. To illustrate these relationships, I show how the number of firms and the level of research per firm (and functions thereof) change across equilibria associated with different values of $c_F$, keeping constant all other parameters and exogenous variables. I use the following parameter values; $\alpha = 0.816$ (implying an elasticity of substitution of 5.4), $\theta = 0.06$, and $m = 6 \cdot 10^{-5}$. For this partial equilibrium analysis, the economy-wide growth rate and interest rate are also held constant at 0.03% and 5%. The calibration strategy used to obtain these values is explained in Appendix A.5.\textsuperscript{16}

Figure 1 illustrates the relationships between the level of research per firm $x$ and the number of firms $N$ implied by the Optimal Research and Free Entry conditions. The Research curve shows the optimal level of research per firm as a function of the number of firms, while the Free Entry curves show the level of research per firm necessary to ensure zero expected profits given the number of firms, for different fixed costs of introducing an innovation. For a given level of research, profits are decreasing in the number firms. At the same time, given the number of firms, profits are decreasing in research. As the number of firms increases, therefore, research must decrease to maintain zero expected profits, resulting in the negative relationship between $x$ and $N$ illustrated by the Free Entry curves. The leftward shift of the Free Entry curve associated with an increase in $c_F$ shows that for a given level of research

\textsuperscript{15}In Section 4, I discuss direct and indirect evidence that supports the relevance of variation in the entry cost of innovators.

\textsuperscript{16}The targets used for the calibration are an industry-level productivity growth rate of 0.3%, an R&D intensity of 2.5%, and a level of competition (as measured in Aghion et al. (2005)) equal to 0.95.
per firm $x$, a higher entry cost means fewer firms can be accommodated under free entry.

![Figure 1: Optimal Research per Firm and Free Entry](image)

The $Research$ curve in Figure 1 illustrates an inverted-U relationship between the number of firms and each firm’s optimal level of research. From the $Research$ condition, each firm’s optimal level of research is clearly independent of $c_F$, except through the effect of $c_F$ on the number of firms $N$. As $N$ increases from one to two, research always increases. Above some threshold $N^*$, however, $x$ decreases with $N$. To understand this relationship, it is useful to note that an increase in the number of firms; (i) decreases each firm’s probability of winning; (ii) increases the expected value of a firm’s innovation, given that it wins; and (iii) decreases the expected markup that the winner can charge. Now consider a variation of the model in which the winning firm can charge a monopoly price regardless of how close its competitors may be. In Appendix A.6, I show the optimal level of firm-level research in a monopoly-only model is both increasing in the number of firms, and always higher than in the full model (except, of course, when the market can only sustain one firm). This is the result of the assumption that the number of firms does not affect the winner’s markup. In the full model, an increase in the number of firms will thus increase the optimal level of
research given monopoly pricing, but make monopoly pricing less likely. The net result is an inverted-U relationship between $x$ and $N$. Note that firm-level research in Figure 1 peaks at around three firms. In general, the location of this peak is weakly increasing in the elasticity of substitution.  

$$c_F = \left[ \frac{(1 + g)^{1-\alpha}(1 - \alpha^N)}{(1 + r)N(N + \alpha)} \right]^{\frac{1}{\alpha \theta}} \left[ \frac{\theta(N + \alpha - 1)}{c_x} \right]^{\frac{\alpha \theta}{1 - \alpha \theta}} [1 - \theta(N + \alpha - 1)]. \quad (2)$$

Figure 2 plots the equilibrium values of $N$ for different values of $c_F$, while Figure 2b does the same for $x$.  

Not surprisingly, free entry ensures the number of firms adjusts downwards as the fixed cost of innovating increases. With $N$ a monotonic function of $c_F$, the qualitative relationship between $x$ and $c_F$ is just the reverse of that between $x$ and $N$. The result is the inverted-U relationship in Figure 2b. Although not illustrated here, the research intensity of

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17 The relationship between $N^*$ and the elasticity of substitution is discussed in Appendix A.4.2. Allowing labor to move freely across markets would result in $x$ peaking with two firms, regardless of the elasticity of substitution.

18 The range of values for $c_F$ has been chosen to generate the range of values for measured competition reported in Aghion et al. (2005). This variation in competition is illustrated in Figure 3.
an industry (the fraction of total output spent on R&D) can be expressed simply as $N \cdot x \cdot c_x$.

While Figure 2b illustrates an inverted-U between firm-level research and $c_F$, industry-level research intensity decreases monotonically from a high of 6% to a low of 1%, as $c_F$ increases over the illustrated range.

To analyze the relationships between productivity growth, innovation, and competition, I first find the equilibrium values of each variable for different values of $c_F$. I then plot these variables against each other, where each point represents the set of equilibrium values associated with a particular $c_F$. The rate of total factor productivity growth in an industry can be expressed as:

$$g = E(h_{i[1]}^{\alpha})^{\frac{1}{\alpha}} - 1 = \left(\frac{N x^{\alpha \theta}}{N + \alpha}\right)^{\frac{1}{\alpha}} - 1.$$  

The average level of innovation per firm is the expected value of each draw:

$$Average\ Innovation = E_t^{-1}\left(\frac{A_{i,t}}{A_{i[1],t-1}}\right) = E(h_i) = \frac{x^\theta}{2}.$$  

Following Aghion et al. (2005), I use one minus the average Lerner Index in an industry as the measure of competition in that industry. The Lerner Index is equal to the ratio of price minus marginal cost over price for the best firm, and zero for all other firms.\footnote{This means measured competition is equal to $1 - E(Lerner)/N$, where Lerner is the Lerner Index for the winning firm and $N$ is the number of innovating firms. A better industry measure might weight each firm’s Index by its market share, so that competition would be measured here as $1 - E(Lerner)$. The choice does not affect any qualitative results as both measures are increasing in the number of firms, and so I use the same measure as Aghion et al. (2005) for the sake of easier comparison. Both measures are derived in Appendix A.3.} The value of this measure of competition for an industry is thus:

$$Competition = 1 - \frac{1}{N}E\left(\frac{Price - Marginal\ Cost}{Price}\right) = \frac{N^2 + \alpha^N - 1}{N^2}.$$  

Figure 3a plots the level of innovation per firm against competition. Each point represents the values of both variables associated with a particular value of $c_F$.\footnote{The range of values used for $c_F$ is the same as that in Figure 2.}
monotonically decreasing in $c_F$, so average innovation per firm has an inverted-U relationship with competition just as does research per firm in Figure 2b.

![Figure 3](image.png)

(a) Firm-Level Innovation  
(b) Industry-Level Productivity Growth

Figure 3: Competition, Innovation, and Growth, with variation in $c_F$

Figure 3b plots industry-level productivity growth against competition as the cost of introducing an innovation varies. Even while firm-level innovation follows an inverted-U, industry-level productivity growth always increases with competition. The decrease in the expected markup of the winning firm due to more competition tends to discourage research, all else equal, since a drop in the fraction of value captured by the winner means firms are less interested in winning the market ex ante—this is the Schumpeterian effect. But competition also tends to increase the expected value of the best innovation — this is the Hayekian effect. At low levels of competition, an increase in competition is associated with more firm-level research, more innovations, and higher growth. At relatively high levels of competition, where the Schumpeterian effect causes firm-level research and average innovation to decline with competition, the Hayekian effect more than compensates, increasing the expected value of the best innovation, and thus increasing industry-level productivity growth. The range of productivity growth rates illustrated in Figure 3b is comparable to the range across 6-digit manufacturing industries in the U.S. Average productivity growth from 1959 to 2005 ranged
from -2% for Other Apparel Manufacturing to +13% for Electronic Computer Manufacturing, with a full one third of industries experiencing a decline in productivity over the period.\footnote{Productivity growth here refers to five-factor productivity growth, as reported in NBER-CES Manufacturing Industry Database.}

The experiment illustrated in Figures 2 and 3 (changing partial equilibrium outcomes by changing $c_F$) can be repeated for each of the exogenous parameters in the model. Given that the number of firms $N$ is a decreasing function of $c_F$ (as illustrated in Figure 2a), equation (2) implies $N$ is decreasing in the marginal cost of research $c_x$, as is research per firm $x$. Both $N$ and $x$ have an inverted-U relationship with the elasticity of substitution. The parameter $\theta$ governs the elasticity of each firm’s upper bound with respect to research, as each firm draws from a distribution with support $[0, x^\theta]$. Research per firm is increasing in $\theta$, while the number of firms is decreasing. An increase in the interest rate or decrease in the economy’s growth rate each lower discounted profits and therefore result in fewer firms and less research per firm in equilibrium.

The effects of economy-wide changes in exogenous parameters and variables (i.e., the effects on variables in general equilibrium) are qualitatively the same as those for the partial equilibrium. The only difference of note is that research (and innovation) per firm is always highest with two firms per market, regardless of the elasticity of substitution. The intuition here is that when the cost of introducing an innovation decreases, the economy-wide growth rate increases and brings the interest rate up with it. The net effect is to decrease expected profits for firms, which puts additional downward pressure on the incentive to do research.

It is worthwhile to digress here to discuss the difference between the mechanism that generates an inverted-U in the present model, and that in Aghion et al. (2005). In Aghion et al., the positive link between competition and innovation (over the initial section of their inverted-U) is driven by neck-and-neck firms innovating more to escape lower rents. With free entry, however, lower rents would presumably lower the incentive to innovate for lagging firms, thus dampening or even wiping out the positive effect of lower rents on the average
level of innovation. Indeed, Etro (2007) finds the “escape competition” effect disappears completely when he incorporates free entry into a number of variants of the model in Aghion et al. In the present paper, free entry itself causes aggregate innovation to increase with competition, due to the Hayekian effect (by increasing the expected value of the best innovation for a given level of research per firm), while also offsetting the disincentive to innovate at the firm level at low levels of competition.

4 Empirical Support

In the preceding section, I focus on the effects of a change in the cost of introducing an innovation $c_F$. This focus is appropriate if changes in $c_F$ are actually driving the relationships reported in empirical studies. In fact, each of the studies cited throughout this paper control for both year and industry fixed effects, and so are presumably reporting relationships driven by within-industry variation in an exogenous parameter over time. The regulatory portion of the fixed cost of introducing an innovation to the market is at least a plausible candidate for the source of this variation. Evidence from Aghion et al. (2005) more directly supports a focus on regulatory costs. In an attempt to control for endogeneity in their test of the effect of competition on firm-level innovation, Aghion et al. employ a series of stuttered waves of deregulation in different industries as a source of exogenous changes to competition. In the end, their IV results are almost identical to their OLS results (controlling for time and fixed effects) with a reduced-form $R^2$ of 0.8, suggesting that much of the correlation between competition and firm-level innovation can be explained by the effect of lower barriers to entry on the number of firms and thus the optimal level of innovation. Variation in the regulatory cost of entry maps well to the fixed cost of introducing an innovation to a market in the model. Another plausible source of the variation in competition in Aghion et al. is variation in import tariffs, which the authors also use as an instrument. This channel is neglected in the present paper, but could be examined in an extension to the model.
The primary implication of the model developed in this paper, that competition and productivity growth are positively related, is well supported in empirical studies of industry-level growth like Nickell (1996), Blundell, Griffith, and Van Reenen (1999), and Aghion, Braun, and Fedderke (2008). To my knowledge, no empirical studies have provided evidence to the contrary. In addition, studies like Graham, Kaplan, and Sibley (1983) and Nicoletti and Scarpetta (2003) have suggested both higher marketing costs and more burdensome regulations are associated with lower growth, which is consistent with the model. Aghion et al. (2005) test the relationship between competition and the average level of innovation per firm and report an inverted-U relationship, consistent with the present model.

In recent publications, economists have interpreted evidence of an inverted-U relationship between firm-level innovation and competition as evidence of a similar relationship between industry-level productivity growth and competition. Aghion, Braun, and Fedderke (2008) present evidence to the contrary. Their results are consistent with those of the model presented here, where variation in the cost of innovating induces positively correlated differences in competition and productivity growth at the industry level, but an inverted-U relationship between competition and firm-level innovation.

Boudreau, Lacetera, and Lakhani (2011) provide evidence of the plausibility of the central mechanism of this paper - that a larger number of innovators will tend to increase the value of the best innovation, notwithstanding the disincentive effect on effort. The authors look at data from 9,661 software algorithm contests in which competitors try to solve a well-defined problem. All proposed solutions are observed and their quality measured. Multiple contests are held for the same problem with exogenous variation in the number of competitors. In contests where the best approach to solving a problem is uncertain, the authors find the quality of the best solution is increasing in the number of competitors, even when the average quality is decreasing.

22See Bianco (2007), for example. Indeed, this view seems to have become the conventional wisdom.
The central mechanism of this paper generates one additional implication. In Appendix A.4.5 I show that the average distance of firms within an industry from the frontier (i.e., the best firm in an industry) is increasing with competition. This is supported by Aghion et al. (2005), who report industries become less ‘neck-and-neck’ as measured competition increases.

The implications of the model with respect to changes in parameters other than fixed costs are broadly consistent with both existing models and the empirical literature on the determinants of R&D, market structure, and productivity growth. Vives (2008) and Etro (2007) provide detailed analyses of the various determinants of R&D and growth for a number of different theoretical models. Cohen and Levin (1989) and Gilbert (2006) provide excellent surveys of empirical studies looking at the effects of differences in appropriability, technological opportunity, and market size.

Finally, note that under the assumption that each product introduction carries the same costs, the number of firms in an industry is indeterminate in Section 2. None of the results in this paper depend on the number of firms in an industry, but it bears mentioning that the firm-level empirical relationships discussed above rely on firm-level measures of innovation and productivity. A direct mapping from the model to these firm-level empirical phenomena therefore requires that a firm’s optimal choice of how many markets to enter be independent of the level of competition in each market. Such would be the case if (for example) the cost of introducing a product were assumed to be increasing in the number of markets entered, so each firm would optimal choose to enter only one market.

5 Extension: Antitrust Policy

In this section, I use the model to consider the effects of antitrust policies on innovation and productivity growth. An advantage of using a model in which competition is endogenous is the ability to specify exactly how various policies affect exogenous constraints or parame-
ters in the model and evaluate the resulting outcomes, rather than merely assume antitrust policy exogenously increases competition. The baseline model developed in Section 2 implies deregulation should increase competition and productivity growth. Where antitrust policy works to lower legal barriers to innovation and market entry, the model predicts a higher aggregate level of innovation will result. In contrast it is straightforward to show that a legislated ceiling on markups (enforced, for example, under ‘Abuse of Dominant Position’ provisions of antitrust legislation) will discourage both entry and innovation, resulting in less growth. In the following subsection, I extend the model to evaluate the effects of legal restrictions on collusion or (equivalently) ‘anti-competitive’ mergers. Whereas empirical studies of competition and productivity growth generally conclude with calls for greater antitrust enforcement (on the presumption that greater enforcement implies more competition, which is found to be positively correlated with growth)23, I find that restrictions on collusion (as with restrictions on markups) lead to lower productivity growth in equilibrium. The finding that restrictions on markups and collusive behavior are detrimental to innovation and growth is consistent with conventional models of innovation. The contribution of this section is to show that even in a model where measured competition is positively related to growth, restrictions on firm behavior nevertheless result in lower growth.

5.1 Collusion and Anti-Competitive Mergers

This section extends the baseline model by allowing the winning firm in any period to either purchase or collude with any other firm that might constrain the winning firm’s pricing decision. Williamson (1968), Mathewson and Winter (1987), and others have considered situations in which price-fixing and mergers thought to be anti-competitive might actually improve allocative efficiency. The possible benefits of joint research ventures with respect to technology diffusion have also been analyzed in studies like Katz (1986). The contribution

\footnote{Examples of this particular policy proposal can be found in the concluding sections of Geroski (1990) and Dutz and Hayri (2000).}
of this section is to evaluate the effects of restrictions on collusion and mergers for which no efficiency defence exists - that is, behavior undertaken by firms that increases profits at the expense of consumers ex post, and provides no benefit to allocative efficiency. I simplify the model by assuming an exogenous common upper bound \( \lambda > 1 \) for the distribution firms draw from, as allowing for each firm to choose its own level of research adds little additional insight.

I model the cost of purchasing or colluding with all firms that pose a threat as an exogenous fraction \( \tau^C \) of the additional profit firm-[1] (the winning firm) stands to gain by charging a monopoly price, rather than a limit price. In addition, I assume firm-[1] must incur a cost to monitor its co-conspirators (or manage its larger size after merging), which I model in a similar fashion as a fraction \( \tau^M \).\(^{24}\) Antitrust policy here can be interpreted as a marginal increase in either of these costs, or simply as an assumption that antitrust enforcement makes either of these costs high enough to make collusion always a bad idea for firms.

The total cost of achieving a monopoly price when \( \frac{h_{[2]}}{h_{[1]}} > \alpha \) is;

\[
(\tau^C + \tau^M) \frac{\psi(h_{[2]} - \alpha h_{[1]})}{h_{[1]}^{1-\alpha}},
\]

where \( \psi \equiv Y^{1-\alpha} A_{[1],-1}^\alpha \), \( A_{[1],-1} \) is the first-best quality of the previous period, and \( h_{[\ell]} \) is the \( \ell \)th-best draw of the current period. The payoff for each firm-[\( r \)] for which \( \frac{h_{[r]}}{h_{[1]}} > \alpha \) is equal to the fraction \( \tau^C \) of the additional profit the winning firm receives by colluding with firm-[\( r \)]. The payoff for an eligible firm-[\( r \)] is therefore;

\[
\tau^C \frac{\psi(h_{[r]} - \max\{h_{[r+1]}, \alpha h_{[1]}\})}{h_{[1]}^{1-\alpha}}.
\]

\(^{24}\)It seems intuitive that each of these costs is increasing with the payoffs to collusion. The larger the payoff to the best firm from colluding, the higher the payment that its rivals could presumably extract. And the further a collusive price is from the competitive limit price, the greater is the incentive for rivals to cheat, which would presumably increase the cost of monitoring.
When $\frac{h_{[2]}}{h_{[1]}} > \alpha$, the operating profits of a winning firm choosing a limit price are $\frac{\psi(h_{[1]} - h_{[2]})}{h_{[1]}}$, while those of a winning firm choosing to collude are $(1 - \alpha)\psi h_{[1]} - (\tau^C + \tau^M)\frac{\psi(h_{[2]} - \alpha h_{[1]})}{h_{[1]}}$. Firm-[1] will therefore choose to collude if $\frac{h_{[2]}}{h_{[1]}} > \alpha$ and;

$$(1 - \alpha)h_{[1]}^\alpha - \frac{h_{[1]} - h_{[2]}}{h_{[1]}^{1 - \alpha}} \geq (\tau^C + \tau^M)\frac{h_{[2]} - \alpha h_{[1]}}{h_{[1]^{1 - \alpha}}},$$

or if $\tau^C + \tau^M \leq 1$. Since the decision to collude or not depends only on $\tau^C$ and $\tau^M$ (whenever $h_{[2]} > \alpha$), I focus on the interesting case where $\tau^C + \tau^M \leq 1$.

I now derive the expected discounted profits facing each innovating firm. Because the payoffs to eligible losing firms are straight transfers and firms have no ability to affect their expected payoff (or payment) ex ante, I need not account for payoffs or payments. The expected discounted value of introducing an improved product for a firm-$i$ is thus;

$$V_i = \frac{(1 - \alpha)\Psi (1 + g)^{1 - \alpha}}{1 + r} \text{Prob} \left[ h_i = h_{[1]} \right] \cdot E \left[ h_{[1]}^\alpha \mid h_i = h_{[1]} \right] - \frac{\Psi \tau^M (1 + g)^{1 - \alpha}}{1 + r} \text{Prob} \left[ h_i = h_{[1]} - \frac{h_{[2]}}{\alpha} \right] \cdot E \left[ \frac{h_{[2]} - \alpha h_{[1]}}{h_{[1]}^{1 - \alpha}} \mid h_i = h_{[1]} < \frac{h_{[2]}}{\alpha} \right] - \Psi (c_F + c_x x_i),$$

where the first term is the expected monopoly profits from production, the second term is the expected cost of monitoring the winning firm’s co-conspirators, and $\Psi$ is defined as in Section 2. Using the density function derived in Appendix A.1 and setting the expected value of entry equal to zero, the collusive equilibrium can now be characterized by the following Free Entry condition;

$$\text{Free Entry: } c_F = \frac{(1 + g)^{1 - \alpha} \lambda^\alpha}{(1 + r)N(N + \alpha)} (N(1 - \alpha) - \tau^M [N(1 - \alpha) + \alpha^N - 1]).$$

Note that if monitoring costs eat up the entire benefit of collusion ($\tau^M = 1$), then this
condition reduces to that of a competitive equilibrium.\textsuperscript{25} Further, the number of innovators is independent of $\tau^C$, conditional on $\tau^C + \tau^M \leq 1$.

The right-hand side of the Free Entry condition is simply the expected discounted operating profits of an innovator divided by a constant ($\Psi$), and so is decreasing in the number of firms $N$. A higher cost of monitoring $\tau^M$ reduces expected profits (given $N$), and so the number of firms must be decreasing in $\tau^M$ under free entry. The growth rate for an industry is $g = \left(\frac{N\lambda\alpha}{N+\alpha}\right)^{\frac{1}{\alpha}} - 1$, which is increasing in $N$. It follows that $g$ must be decreasing in $\tau^M$. Given that the growth rate for a collusive industry is equal to that in a non-collusive industry when $\tau^M$, it must be the case that growth is higher if collusion is possible, as long as $\tau^M < 1$.

Testing these results against real outcomes is difficult, due to the ubiquitousness of antitrust laws throughout the last century. The story of Standard Oil in the late nineteenth and early twentieth centuries, however, does provide a case study that seems consistent with the implications of the present model.\textsuperscript{26} Over a forty-year period, the company used a combination of process and product innovations, acquisitions, and price-fixing agreements to dominate the market, maintaining a market share of close to 90\% until both politics and more able rivals began to drag them back down to earth. Just as the model predicts, a large number of new entrants appeared each year, many seemingly with the intention of becoming just competitive enough to get bought-out by Standard Oil. While some refineries were used to add to capacity, many were just bought and then shut down in an effort to reduce competition. Nevertheless, the period saw an enormous number of new process innovations, as well as new ways to turn the ‘waste’ from the production of kerosene into products like gasoline, paving tar, and petroleum jelly. Over the course of thirty years, just as Standard Oil first acquired and then maintained its monopoly, the deflation-adjusted price of kerosene dropped by over 65\%, even while petroleum output increased by more than a factor of three.

\textsuperscript{25}For the purposes of this section, I refer to the equilibrium in Section 2 as the ‘competitive equilibrium’.
\textsuperscript{26}McGee (1958) provides a detailed analysis of the competitive behavior of Standard Oil and its rivals. Boudreaux and Folsom (1999) provide a short summary of the innovations and price reductions that accompanied J.D. Rockefeller’s entertaining quest to dominate the market.
Finally it is important to point out that the model is ill-suited for evaluating the welfare implications of collusion, as it does not allow for any static inefficiency resulting from a monopoly price (since labor is immobile). That collusion may be welfare-reducing even while increasing growth is most obvious in the limiting case where monitoring costs erode the entire benefit of charging a monopoly price. When $\tau^M$ is equal to one, growth will be no higher than in a competitive industry, but resources will nonetheless be wasted enforcing the collusive agreement. The policy implication of this model is that regulators should either allow price-fixing agreements to be enforced, thereby lowering monitoring costs, or else raise monitoring costs to a prohibitive level. The appropriate choice depends on the trade-off between dynamic and static efficiency (as well as on enforcement costs and ideology).

6 Conclusion

Hayek (2002) argued that the competitive process could be thought of as a procedure for discovering and making use of knowledge that would otherwise not emerge. In this paper, competition works to discover which quality improvements are most highly demanded by the consumers of innovations. When firms are uncertain about the optimal direction of innovation, the best innovation to emerge will tend to be of higher value when more innovations are tried. Throughout this paper the Schumpeterian mechanism, whereby lower ex post rents discourage lower ex ante research, is always present. But when Schumpeter meets Hayek in this model of innovation, more competition is accompanied by higher industry-level productivity growth, even if research per firm is lower. Combining Hayekian uncertainty with endogenous entry, markups, and firm-level research results in a model able to generate a positive relationship between competition and industry-level productivity growth, and an inverted-U relationship between competition and firm-level innovation.

By treating competition, growth, and innovation as endogenous, I develop a framework for evaluating explicitly-modeled antitrust policies. Two such evaluations of antitrust poli-
cies (restrictions on markups and price-fixing) come to conclusions very much at odds with policy prescriptions based on simple interpretations of empirical studies. The results of recent empirical studies notwithstanding, I conclude that one of Schumpeter’s central messages should be taken to heart — regardless of any static benefits of antitrust enforcement and regulation, these policies come with the cost of less growth. Or alternatively, there ain’t no such thing as a free lunch.

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A Appendix

A.1 Density Functions

To solve the model, it is necessary to derive the joint density function of \( h_1, h_2, \) and \( h_i \). If \( h_\ell \) is distributed according to a continuously differentiable distribution \( F_\ell(h) \) between 0 and \( x_\ell^\theta \), the relevant joint density function is:

\[
f(h_1 = u_1, h_2 = u_2, h_i = h_1 \mid N, x_i, x_k \neq i) = 
\int_{0}^{u_2} \cdots \int_{0}^{u_{N-1}} (N-1)! f(h_i = u_1) \prod_{\ell=2}^{N} f(h_k = u_\ell) du_N \cdots du_3,
\]

where \( N \) is the number of firms (draws), \( x_i \) is firm-\( i \)’s level of research, and \( x_k \) is every other firm’s level of research. Integrating over all \( u_\ell > 2 \), this simplifies to:

\[
f(h_1 = u_1, h_2 = u_2, h_i = h_1 \mid N, x_i, x_k \neq i) = (N-1) f_i(u_1) f_k(u_2) F_k(u_2)^{N-2},
\]

defined over all \( u_1 \) and \( u_2 \), such that \( u_1 \leq x_\ell^\theta \), \( u_2 \leq x_k^\theta \), and \( u_1 \geq u_2 \geq 0 \). If each firm-\( \ell \)’s quality draw is from a uniform distribution bounded by 0 and \( x_\ell^\theta \), then \( f_\ell(u) = \frac{1}{x_\ell^\theta} \) and \( F_\ell(u) = \frac{u}{x_\ell^\theta} \). It follows that the above joint density function becomes:

\[
f(h_1 = u_1, h_2 = u_2, h_i = h_1 \mid N, x_i, x_k \neq i) = \frac{(N-1) u_2^{N-2}}{x_i^\theta x_k^{g(N-1)}}.
\]

When calculating the expected value of an equilibrium variable, the relevant joint density function is similar to the one above, but without the qualification that firm-\( i \) has the highest draw. Taking into account \( x_i = x_k = x \) in equilibrium, the required density function is:

\[
f(h_1 = u_1, h_2 = u_2 \mid N, x) = \int_{0}^{u_2} \cdots \int_{0}^{u_{N-1}} N! \prod_{\ell=1}^{N} f(h = u_\ell) du_N \cdots du_3,
\]
or

\[ f(h_{[1]} = u_1, h_{[2]} = u_2 \mid N, x) = N(N - 1)f(u_1)f(u_2)F(u_2)^{N-2}, \]

defined over all \( u_1 \) and \( u_2 \), such that \( x^\theta \geq u_1 \geq u_2 \geq 0 \). If each firm draws from the same uniform distribution, this joint density function becomes;

\[ f(h_{[1]} = u_1, h_{[2]} = u_2 \mid N, x) = \frac{N(N-1)u_2^{N-2}}{x^\theta N}. \]

**A.2 Equilibrium**

Using the joint density function from Appendix A.1, the expected discounted value of introducing an improved product (equation (1)) can be expressed as:

\[
V_{i,t-1} = \frac{\Psi_{t-1}(1 + g)^{(1-\alpha)(1-\alpha)}N}{1 + r} \int_0^{\theta_{i,t-1}} \int_0^{\theta_{i,t-1}} \frac{(N_{t-1} - 1)u_1^\alpha u_2^{N_{t-1} - 2} \theta_{i,t-1}^{\theta_{i,t-1} - 1}}{x_i^{\theta_{i,t-1}} x_k^{\theta_{i,t-1}}} du_2 du_1 \\
+ \frac{\Psi_{t-1}(1 + g)^{(1-\alpha)}N}{1 + r} \int_0^{\theta_{i,t-1}} \int_0^{\theta_{i,t-1}} \frac{(N_{t-1} - 1)(u_1 - u_2)u_2^{N_{t-1} - 2} \theta_{i,t-1}^{\theta_{i,t-1} - 1}}{x_i^{\theta_{i,t-1}} x_k^{\theta_{i,t-1}}} du_2 du_1 \\
- \Psi_{t-1}(c_F + c_x x_{i,t-1}).
\]

Taking for granted that decisions are time-independent, this value function can be expressed as;

\[
V_i = \frac{\Psi(1 + g)^{(1-\alpha)(1-\alpha)}N}{(1 + r)N(N + \alpha)x_i^{\theta(N+\alpha-1)}} - \Psi(c_F + c_x x_i).
\]

Using \( V_i \) with equilibrium conditions (iv) and (v) results in the *Optimal Research* and *Free Entry* conditions.

The growth rate of the economy \( g_t \) is equal to;

\[
g_t = \frac{Y_t}{Y_{t-1}} - 1 = \left( \frac{\int_0^1 A_{[1],m,t-1}^{\alpha} h_{[1],m,t}^{\alpha} dm}{\int_0^1 A_{[1],m,t}^{\alpha} h_{[1],m,t}^{\alpha} dm} \right)^{\frac{1}{\alpha}} - 1.
\]

\(^{27}\)To be precise, the above is true only if \( x_{i,t-1} \leq x_{k,t-1} \). But if one were to instead assume \( x_{i,t-1} \geq x_{k,t-1} \), the same *Free Entry* and *Optimal Research* conditions would result in equilibrium, where \( x_{i,t-1} = x_{k,t-1} \).
Given arbitrary $A_{m,t=0}$, $g_t$ can be expressed as;

$$g_t = \left( \frac{\int_0^1 A_{m,0} \prod_{s=1}^{t} h_{[1],m,s} \, dm}{\int_0^1 A_{m,0}^{\alpha}(j) \prod_{s=1}^{t-1} h_{[1],m,s} \, dm} \right)^{\frac{1}{\alpha}} - 1.$$ 

In a symmetric economy where all parameter values including $A_0$ are common across markets and industries, the above expression is equal to;

$$g_t = \left( \frac{E\left[ \prod_{s=1}^{t} h_{[1],s}^{\alpha} \right]}{E\left[ \prod_{s=1}^{t-1} h_{[1],s}^{\alpha} \right]} \right)^{\frac{1}{\alpha}} - 1.$$ 

With independence of draws across time periods, $g_t$ can be expressed as;

$$g_t = \left( \frac{E\left[ h_{[1]}^{\alpha} \right]^{t}}{E\left[ h_{[1]}^{\alpha} \right]^{t-1}} \right)^{\frac{1}{\alpha}} - 1 = E \left[ h_{[1]}^{\alpha} \right]^{\frac{1}{\alpha}} - 1.$$ 

Using the joint density function derived in Appendix A.1, this becomes;

$$g = \left( \int_0^{x^\theta} \int_0^{u_1} \frac{N(N-1)u_1^\alpha u_2^{N-2}}{x^\theta N} \, du_2 \, du_1 \right)^{\frac{1}{\alpha}} - 1 = \left( \frac{Nx^\alpha}{N + \alpha} \right)^{\frac{1}{\alpha}} - 1.$$ 

**A.3 Measuring Competition**

Competition (as measured in Aghion et al. (2005)) in an industry with a continuum of product markets is equal to;

$$1 - \frac{1}{N} E \left( \frac{\text{Price} - \text{Marginal Cost}}{\text{Price}} \right).$$
From the firm’s problem, the optimal markup in market $m$ is such that \( \frac{P_m - MC_m}{P_m} \) is equal to;

\[
\frac{P_m - MC_m}{P_m} = \begin{cases} 
1 - \alpha, & \text{if } \frac{h[2,m]}{h[1,m]} < \alpha \\
1 - \frac{h[2,m]}{h[1,m]}, & \text{if } \frac{h[2,m]}{h[1,m]} > \alpha.
\end{cases}
\]

Competition in an industry with a continuum of product markets is therefore;

\[
\text{Competition} = \frac{N - 1}{N} + \int_0^u \int_0^{\alpha u_1} \frac{\alpha}{N} f(h[1] = u_1, h[2] = u_2 \mid N, x) du_2 du_1 \\
+ \int_0^u \int_0^{u_1} \frac{u_2}{u_1 N} f(h[1] = u_1, h[2] = u_2 \mid N, x) du_2 du_1.
\]

Substituting in the density function from Appendix A.1 results in;

\[
\text{Competition} = \frac{N^2 + \alpha^N - 1}{N^2}.
\]

An alternative measure of competition might give weight only to the Lerner Index of producing firms, so that competition would be measured as \( 1 - E(\text{Lerner}) \), or;

\[
\text{Competition (alternative)} = 1 - E\left( \frac{\text{Price} - \text{Marginal Cost}}{\text{Price}} \right) = \frac{N + \alpha^N - 1}{N}.
\]

To see that this alternative measure of competition is increasing in $N$, note that;

\[
\frac{\partial}{\partial N} \left( \frac{N + \alpha^N - 1}{N} \right) = \frac{\alpha^N \ln(\alpha^N)}{N^2} + \frac{1 - \alpha^N}{N^2},
\]

which is greater than zero for all $N > 0$, $\alpha \in (0, 1)$.
A.4 Proofs of Results

For convenience, I repeat the Optimal Research and Free Entry conditions;

\[ \text{Optimal Research: } x = \left[ \frac{(1 + g)^{1-a}(1 - \alpha^N)\theta(N + \alpha - 1)}{c_x(1 + r)N(N + \alpha)} \right]^{\frac{1}{1-a}} \]

\[ \text{Free Entry: } c_F + c_xx = \frac{(1 + g)^{1-a}(1 - \alpha^N)x^{\alpha\theta}}{(1 + r)N(N + \alpha)}. \]

A.4.1 Number of Firms Decreasing in \( c_F \)

I start by showing that the equilibrium number of firms is decreasing in the fixed cost of introducing an innovation \( c_F \). The right-hand side of the Free Entry condition is simply the expected discounted operating profits of an innovating firm (divided by a constant), and so must be decreasing in the number of firms \( N \), given \( x \). Since \( x \) maximizes expected profits, any increase in \( x \) (given \( N \)) must increase \( c_xx \) more than it increases expected profits. Since the left-hand side of the Free Entry condition increases more than the right-hand side when \( c_F \) and \( x \) increase, the number of firms must decrease to satisfy the condition. Again because \( x \) is maximizing expected profits, any decrease in \( x \) (given \( N \)) must decrease expected profits more than it decreases \( c_xx \). In this case, too, \( N \) must drop to satisfy the Free Entry condition.

A.4.2 Average Innovation Has Inverted-U Relationship with \( c_F \)

\( c_F \) does not enter into the Optimal Research condition, and so research per firm \( x \) only depends on \( c_F \) through its effect on \( N \). Since \( N \) is monotonically decreasing in \( c_F \), I simply focus on the relationship between \( x \) and \( N \). It is clear from the Research condition than \( x \) is increasing if \( (1-\alpha^N)/(N+\alpha-1) \) is increasing, and decreasing otherwise (when \( N \) is changing as a result of a change in \( c_F \)). Notice the numerator \( (1 - \alpha^N)(N + \alpha - 1) \) is increasing in \( N \) at a decreasing rate (since the rate of increase for \( (N + \alpha - 1) \) is constant and \( \frac{\partial^2}{\partial N^2} (1 - \alpha^N) = -\alpha^N \ln(\alpha)^2 < 0 \), and the denominator is increasing in \( N \) at an increasing rate. It follows
that there exists some $N^*$, such that research is decreasing in $N$ for all $N > N^*$. To prove that $x$ has an inverted-U relationship with $N$, I further show that research per firm is always higher when $N = 2$ than when $N = 1$:

$$x(N = 2) = \phi \left[ \frac{(1 - \alpha^2)(1 + \alpha)}{2(2 + \alpha)} \right]^{\frac{1}{1-\alpha \theta}} > \phi \left[ \frac{(1 - \alpha)\alpha}{(1 + \alpha)} \right]^{\frac{1}{1-\alpha \theta}} = x(N = 1),$$

for all $\alpha \in (0, 1), \theta \in (0, 1)$. By solving $\frac{\partial}{\partial N} \left( \frac{(1-\alpha^N)(N+\alpha-1)}{N(N+\alpha)} \right) = 0$ for $N$, I can also show that for values of $\alpha$ less than 0.5 (implying an elasticity of substitution equal to 2), $N^*$ is equal to about 2. As $\alpha$ increases from 0.5, $N^*$ also increases, as shown in Figure 4. Finally, note that average innovation per firm is a simple increasing function of research per firm, and so its relationships with $N$ and $c_F$ are the same as those for $x$.

![Figure 4: $N^*$: Number of Firms at which Research per Firm Peaks](image)

**A.4.3 Competition Decreasing in $c_F$**

To show measured competition is decreasing in $c_F$, I take for granted that $N$ is decreasing in $c_F$, and show competition is increasing in $N$. Measured competition for an industry with
innovating firms is;
\[ \frac{N^2 + \alpha^N - 1}{N^2} = 1 - \frac{1 - \alpha^N}{N^2}. \]

Using the same argument made above, there must exist some \( N^* \) such that competition is increasing in \( N \) for all \( N > N^* \). To prove competition is always increasing in \( N \) (for all \( N \geq 1 \)), I further show that competition is always higher when \( N = 2 \) than when \( N = 1 \);

\[
\text{Competition} (N = 2) = 1 - \frac{1 - \alpha^2}{2} > \alpha = \text{Competition} (N = 1).
\]

A.4.4 Growth Decreasing in \( c_F \)

Denoting the industry-level growth rate as \( g \), 1 + \( g = \left( \frac{N x^\theta}{N + \alpha} \right) \frac{1}{\alpha} \). To prove growth is decreasing in \( c_F \), given that the number of firms \( N \) is decreasing in \( c_F \), I show that;

\[
\frac{d[(1 + g)^\alpha]}{dN} = \frac{\partial[(1 + g)^\alpha|N]}{\partial N} + \frac{\partial[(1 + g)^\alpha|N]}{\partial x} \cdot \frac{\partial x(N)}{\partial N} > 0.
\]

Using the Optimal Research condition above to obtain \( x \) as an explicit function of \( N \), the above inequality becomes;

\[
\frac{\alpha x^{\alpha \theta}}{(N + \alpha)^2} + \frac{N \theta \alpha x^{\alpha \theta}}{N + \alpha} \cdot \frac{\partial x(N)}{\partial N} > 0,
\]

which holds if \( \frac{x}{N \theta (N + \alpha)} > - \frac{\partial x(N)}{\partial N} \). The partial derivative \( \frac{\partial x(N)}{\partial N} \) is equal to;

\[
\frac{\partial x(N)}{\partial N} = \frac{x^{\alpha \theta}(1 + g_Y)^{1-\alpha}}{c_x(1 + r)} \cdot \frac{-\alpha^N \ln(\alpha^N) + (1 - \alpha^N)((1 - \alpha)(2N + \alpha) - N^2)}{N^2(\alpha^N)^2},
\]

where \( g_Y \) is the economy-wide growth rate (held constant while \( c_F \) is varied for only one industry). Using the expression for \( x(N) \) given by the Research condition, the inequality
that needs to hold can now be expressed as;

$$\frac{(1 - \alpha^N)(N + \alpha - 1)}{\theta} > \alpha^N \ln(\alpha^N) - (1 - \alpha^N)[(1 - \alpha)(2N + \alpha) - N^2].$$

Note that for any given $N$ and $\alpha$, the left-hand side of the inequality is decreasing in $\theta$. Assume for the moment that $\theta$ is constrained to be no higher than $\frac{1}{N+\alpha-1}$. It would then be sufficient to show that the inequality holds for $\theta = \frac{1}{N+\alpha-1}$. Under this assumption, the inequality becomes:

$$(1 - \alpha^N)(N + \alpha - 1)^2 > \alpha^N \ln(\alpha^N) - (1 - \alpha^N)[(1 - \alpha)(2N + \alpha) - N^2],$$

or

$$(1 - \alpha^N)(1 - \alpha) > \alpha^N \ln(\alpha^N),$$

which holds for all $\alpha \in (0, 1)$ and $N \geq 1$. To see that $\theta$ can be no higher than $\frac{1}{N+\alpha-1}$, combine the Free Entry and Optimal Research conditions to obtain:

$$c_F = \frac{c_x x [1 - \theta(N + \alpha - 1)]}{\theta(N + \alpha - 1)}.$$

This expression can be interpreted as characterizing the value of $c_F$ needed to accommodate an equilibrium with a number of firms equal to $N$ and a level of research per firm equal to $x$, given all other parameter values. Assuming $c_F \geq 0$ it is clear that $\theta$ can be no higher than $\frac{1}{N+\alpha-1}$, for any given $N$.

A.4.5 Average Distance to Frontier Increasing with Competition

To show that the distance between the quality of the average firm and the best firm is increasing with competition, I need to relax the assumption that each industry contains a continuum of product markets. With a continuum of markets, the best firm in an industry
would always have a quality equal to the upper bound of the distribution from which firms draw, irrespective of the number of draws in each market. Consider an industry made up of \( M \) product markets, with \( N \) innovating firms in each market. The expected gap between the average and the best firm within this industry is;

\[
\text{gap} = E \left( \frac{h_{[1]} - \frac{1}{N \cdot M} \sum_{\ell=1}^{N \cdot M} h_{[\ell]}}{h_{[1]}} \right) = 1 - \frac{1}{N \cdot M} \sum_{\ell=1}^{N \cdot M} E \left( \frac{h_{[\ell]}}{h_{[1]}} \right),
\]

where \( h \sim U(0, x^\theta) \) as before, and \( h_{[\ell]} \) now denotes the \( \ell^{th} \)-best of \( N \cdot M \) draws. Using the same steps followed in Appendix A.1, one can derive the expected value of \( \frac{h_{[\ell]}}{h_{[1]}} \):

\[
E \left( \frac{h_{[\ell]}}{h_{[1]}} \right) = \frac{N \cdot M - \ell + 1}{N \cdot M},
\]

which results in an expected gap equal to;

\[
\text{gap} = 1 - \frac{1}{N \cdot M} \sum_{\ell=1}^{N \cdot M} \left( \frac{N \cdot M - \ell + 1}{N \cdot M} \right) = \frac{1}{2} \left( 1 - \frac{1}{N \cdot M} \right),
\]

which is increasing in \( N \) for any finite \( M \). Since measured competition is also increasing in \( N \), it follows that the gap between the average and the best firm in an industry is increasing in competition.

### A.5 Calibration

To illustrate the relationships generated by the model in Section 3, I need values for \( \alpha, \theta, \) and \( c_x \), as well as the economy-wide growth rate and interest rate. I obtain these values (as well as a value for \( c_F \)) by calibrating the model to match the following targets; industry-level productivity growth of 0.3%, an R&D to output ratio of 0.025, a level of competition (as measured in Aghion et al. (2005)) equal to 0.95, and a real interest rate of 5%. I further ensure that the number of firms are such that research per firm is at its maximum (given
all other parameter values). The productivity growth target is the median industry-level growth rate of five-factor-productivity across 473 6-digit NAICS manufacturing industries in the U.S., where each industry’s growth rate is an average of annual growth rates from 1959 to 2005 reported in the NBER-CES Manufacturing Industry Database. R&D intensity is from the National Science Foundation. Aghion et al. (2005) report a median level of measured competition for their sample equal to 0.95, and further report the peak of their inverted-U between firm-level innovation and competition as occurring at the median value of competition. I derive the equilibrium values of these variables as I discuss them in Section 3. Through this procedure I obtain the following values: $\alpha = 0.816$ (implying an elasticity of substitution of 5.4), $\theta = 0.06$, $c_F = 6 \cdot 10^{-5}$, and $c_F = 0.04$. In Figures 1 through 3, I illustrate the effects on equilibrium variables from varying $c_F$ (around the calibrated value) for one industry while keeping the interest rate and growth rate of the economy fixed. The growth rate of an economy with a representative industry and a population of 1 is identical to the industry-level productivity growth rate, so I assume the growth rate of the economy is also equal to 0.3%. Finally, I choose the range of $c_F$ to ensure the model generates a range in measured competition from 0.87 to 0.995, which is the range reported for the sample of industry-years in Aghion et al. (2005).

A.6 Monopoly-Only

Here I consider each firm’s optimal level of research when the winning firm can always charge a monopoly price, regardless of how close its competitors may be.

The expected discounted value of introducing an improved product for a firm-$i$ is now:

$$V_i = \frac{\Psi(1 + g)^{1-\alpha}(1 - \alpha)}{(1 + r)} \text{Prob}[h_i = h_{[1]}] \cdot E[h_{[1]}^\alpha | h_i = h_{[1]}]$$

$$- \Psi(c_F + c_x x_i).$$
Using the joint density function from Appendix A.1, this value function can be expressed as:

\[
V_i = \frac{\Psi(1 + g)^{1-\alpha}(1 - \alpha)}{(1 + r)} \int_{x_i}^{x_i^\theta} \int_0^{u_1} \frac{(N - 1)u_1^\alpha u_2^{N-2}}{x_i^\theta x_k^\theta(N-1)} du_2 du_1 - \Psi(c_F + c_x x_i),
\]

or

\[
V_i = \frac{\Psi(1 + g)^{1-\alpha}(1 - \alpha)x_i^{\theta(N+\alpha-1)}}{(1 + r)(N + \alpha)x_k^{\theta(N-1)}} - \Psi(c_F + c_x x_i).
\]

Each firm-\(i\) chooses its level of research \(x_i\) to maximize \(V_i\), given \(x_{k\neq i}\) and \(N\), so the optimal level of research must satisfy:

\[
x = \left[\frac{(1 + g)^{1-\alpha}(1 - \alpha)\theta(N + \alpha - 1)}{c_x(1 + r)(N + \alpha)}\right]^\frac{1}{1-\alpha^\theta}.
\]

Comparing the above equation to the Optimal Research condition in Section 2, research in the monopoly-only model \(x^M\) is always higher than research in the full model \(x^C\), given a number of firms greater than one:

\[
x^M = \phi(N)(1 - \alpha)^\frac{1}{1-\alpha^\theta} > \phi(N)\left(\frac{1 - \alpha^N}{N}\right)^\frac{1}{1-\alpha^\theta} = x^C,
\]

where \(\phi(N) \equiv \left[\frac{(1 + g)^{1-\alpha}\theta(N + \alpha - 1)}{c_x(1 + r)(N + \alpha)}\right]^\frac{1}{1-\alpha^\theta}\). Further, \(x^M\) is always increasing in the number of firms:

\[
\frac{\partial x^M}{\partial N} = \frac{(x^M)^{\frac{\alpha^\theta}{1-\alpha^\theta}}(1 + g)^{1-\alpha}(1 - \alpha)\theta}{(1 - \alpha^\theta)c_x(1 + r)(N + \alpha)^2} > 0.
\]
References


