Trend-Cycle Correlation, Drift Break and the Estimation of Trend and Cycle in Canadian GDP

by

Arabinda Basistha*

(West Virginia University)

Abstract:

Recent research on univariate correlated trend cycle models shows that the estimated results are highly sensitive to the specifications of breaks in the data. This paper argues, using a series of Monte Carlo experiments, that bivariate correlated unobserved components (UC) framework with breaks delivers substantially more accurate results for the trend and cycle parameters than the corresponding univariate frameworks in a finite sample size resembling post war data. The paper estimates stochastic trend and cyclical fluctuations in Canada from a bivariate, correlated UC model with drift breaks. In contrast to the univariate results, bivariate estimation shows a fairly volatile stochastic trend after accounting for the drift break along with a negative trend-cycle shock correlation. The estimated cyclical component is moderately large, persistent and consistent with ECRI denoted Canadian recessions.

This draft: 05.09.06.

Key Words: Stochastic trend; Inflation; GDP; Unobserved components model.

JEL Classification: E31, E32, E50.

---

* Correspondence: Arabinda Basistha, PO Box 6025, Dept. of Economics, College of Business and Economics, West Virginia University, Morgantown, WV-26506. Email:arbasistha@mail.wvu.edu.
The author would like to thank Srobona Mitra, Charles Nelson, Tara Sinclair, Dick Startz, two anonymous referees and participants of SNDE and Midwest Macro meetings for helpful suggestions. Usual disclaimers apply.

1. Introduction

Estimation of cyclical fluctuations using unobserved components (UC) model, introduced by Harvey (1985), Watson (1986), Clark (1987), has the advantage of not imposing apriori smoothness (like time trend or Hodrick-Prescott (hereafter HP) (1997)) on the stochastic trend component. However, these models imposed a restriction of zero correlation between trend and cycle shocks. In a recent paper, Morley, Nelson and Zivot (2003) (hereafter MNZ) showed that the estimates of the cycle component is sensitive to the correlation of the trend and cycle shocks. A highly negative correlation of the shocks, as they estimate from the data, results in small cycles that resemble the cyclical component from univariate Beveridge and Nelson (1981) decomposition. Recent research by Perron and Wada (2005) (hereafter PW) shows that the MNZ result of a volatile stochastic trend disappears if one allows for a drift break in the trend.

The idea of using inflation along with output in the UC models for getting better estimates of the cycle was introduced by Kuttner (1994), followed by Apel and Jansson (1999) in the US context, Gerlach and Smets (1997) in a multiple country context, and Kichian (1999) in the Canadian context. Multivariate modeling of the output gap with Canadian data has been used by St. Amant and Van Norden (1997). Multivariate UC models aiming to estimate US cycles with correlated shocks have been used by Basistha and Nelson (2005), and Sinclair (2005). The correlated state-space models have also been used in a different context by Balke and Wohar (2002) to analyze stock price movements.

Using the two new developments in the field — correlated trend-cycle shocks and drift breaks in the stochastic trend — this paper estimates trend and cycle in Canadian GDP building on Kuttner's bivariate inflation–output framework. It argues, using a series of Monte Carlo experiments that allows for breaks, that bivariate correlated UC framework delivers more accurate estimates of the trend and cycle parameters than the univariate frameworks in a finite sample size resembling post-war data. Estimates using Canadian data produce moderately large and persistent cycles with negative trend-cycle correlation and a volatile stochastic trend even after allowing for drift breaks.

The rest of the paper is organized into four sections. The next section uses Monte Carlo experiments to examine the advantages of bivariate modeling. Section 3 lays out and estimates the bivariate model with Canadian data. Section 4 introduces a random walk model of the drift instead of structural breaks in the drift and re-estimates the UC model of Section 3 for robustness. Section 5 concludes.

2. The Monte Carlo Simulations to Examine the Usefulness of a Bivariate Set-up

Monte Carlo experiments are one way to examine how useful an additional indicator is for estimating an unobserved components model. Before setting up the bivariate model of measuring cyclical fluctuations in Canada, Monte Carlo experiments are used to simulate and examine a similar model with artificial data. It assumes there are two observed variables, y_1 and y_2, resembling output and inflation respectively. The variable y_1 at time t is a sum of two unobserved components, x_1 and z.

1. $y_{1,t} = x_{1t} + z_t$

The variable y_2 also depends on two unobserved components; x_2 and the common factor z, with ϕ as the factor loading parameter.

2. $y_{2,t} = x_{2,t} + \phi z_t$

Let x_1 be the stochastic trend component following a random walk with a constant drift μ.

3. $$x_{1,t} = \mu + x_{1,t-1} + \varepsilon_{1,t}$$

Let z be a mean-zero second order autoregressive process – resembling a cycle.

4. $$z_t = \theta_1 z_{t-1} + \theta_2 z_{t-2} + \varepsilon_{z,t}$$

The last component, depends on the first lag of y_2 (the more commonly used inflation specification):

5. $$x_{2t} = \beta_0 + \beta_1 y_{2,t-1} + \varepsilon_{2,t}$$

In generating the data using equations 1 – 5, the correlation coefficients of the inflationary shock with the other two shocks were zero, i.e. $\rho_{1,2} = \rho_{z,2} = 0$. Three values of the trend-cycle correlation are used, strongly negative as in MNZ ($\rho_{1z} = -0.79$), zero as in Watson (1986) and Clark (1987), and strongly positive ($\rho_{1z} = 0.79$). However, in estimating the model, whether univariate or bivariate, all the shocks are allowed to be correlated.

The values of the standard deviation of the three shocks for data generation are $\sigma_1 = 0.9, \sigma_z = 0.71, \sigma_2 = 1.0$. The trend drift (μ) is 0.8, the common factor loading coefficient (ϕ) is 0.4. The autoregressive coefficients, θ_1 and θ_2, are 1.6 and -0.75 respectively. The values of the parameters β_0 and β_1 are 1.5 and 0.5. These values fairly closely resemble the estimates for Canada and the US as reported in Kuttner (1994), MNZ, Kichian (1999).

Finally, one additional specification of the drift term in the stochastic trend is introduced. As PW argues a drift break is important for estimates of trend and cycle, the drift term μ is allowed to have a known break after 50 data points. The break date roughly corresponds to 1973. So the alternative specification of the stochastic trend equation becomes:

6. $$x_{1,t} = \mu + \mu_1 D_t + x_{1,t-1} + \varepsilon_{1,t}$$

where D_t is a dummy variable that takes the value of one after 50 data points. Note that equation 6 is identical to equation 3 when μ_1 is zero.

Before getting into estimation of the above models, a brief discussion of the identification issues is relevant at this point. Equations 1, 3 and 4 represent the univariate model with correlated shocks. The variance-covariance matrix of the shocks is identified even with correlated shocks if the dynamics of the stationary component is rich enough, specifically at least a second order autoregressive process. The proof of this identification scheme is reported in MNZ and Oh and Zivot (2006). Similarly, equations 1 - 5 form the bivariate correlated model with a common component. The number of unknowns in the variance-covariance matrix is six. However, as Schleicher (2003) and Sinclair (2005) show, the number of unknown parameters that can be identified with two series and a second-order autoregressive process is 11. In this sense, the variance-covariance matrix of the shocks in the bivariate system is overidentified[1].

Initially, the data is generated using the bivariate model, equations 1 – 5, i.e. μ_1 is zero. The state space model with the simulated data is estimated thrice – once using equations 1, 3 and 4 (the univariate without break framework), then using equations 1, 6 and 4 (the univariate with break framework) and finally using equations 1 – 5 (the

[1] Appropriate procedures to test this overidentification should be possible to develop but not pursued in this paper.

bivariate framework). The sample size for each Monte Carlo simulation is 200 – roughly the size of post war quarterly data. The number of Monte Carlo iterations is 3000.

The results of the above Monte Carlo experiments are in Table 1. For each panel, ‘No Break’ means the univariate estimation was done using equation 3 as the stochastic trend whereas ‘Break’ implies the univariate estimation was done using equation 6 as the stochastic trend. Panel A shows the results with negative trend-cycle correlation in the artificial data. The univariate ‘No Break’ mean estimates of the standard deviation of the trend shock, standard deviation of the cycle shock and the correlation of the trend and the cycle shocks from are moderately close to the actual data. But these are fairly imprecise – the standard deviations (reported in the parentheses) are fairly large. Estimates from the ‘Break’ model shows a worse result. The estimates are further away from their actual values with larger standard deviations. The correlation coefficient of the trend cycle shock is nearly zero. However, the distribution of that coefficient is bimodal with a strong tendency to converge to either -1 or 1, especially 1. This property of the ‘Break’ model is present throughout all the Monte Carlo experiments done in the paper. The estimates of the drift terms are accurate in the univariate models.

The bivariate model, on the other hand, yields a striking improvement over the univariate models. The mean estimates of the standard deviation of the trend shock, standard deviation of the cycle shock and the correlation of the trend and the cycle shocks are very close to actual data – and much more precise than the univariate model as measured by the standard deviations. The rest of the parameters are very close to their actual values as well[2]. This result, that the bivariate model yields a better result, is not

[2] The result is robust to inclusion of a break in the drift of the stochastic trend, i.e., using equation 6 as the stochastic trend instead of equation 3 in the bivariate model.

surprising; given that it is the more complete model. The result that the magnitudes of improvements are fairly large for this sample size is the more interesting feature of the simulation results[3].

Panel B and Panel C report the results of the same experiment with different values of the trend cycle correlation. We see a similar pattern as in Panel A. The bivariate results are always more accurate and precise than their univariate counter parts in correctly estimating the true value of the correlation coefficient – a key parameter in estimating the cyclical component, as MNZ demonstrates. The univariate 'Break' model in Panel C does present an accurate trend cycle correlation coefficient but with a lot of estimates converging to one. In general, Table 1 illustrates the observation that MNZ and PW observe – allowing for a drift break in the univariate models makes a fairly large difference in the estimates, even though both are correct models for output dynamics under this data generation scheme. It also shows that using a bivariate setup may help to understand true picture in a dataset with finite sample size.

Two further sets of Monte Carlo experiments were conducted. In the first, the data is generated using equation 6 as the stochastic trend. The value for μ is 1.2 and μ_1 is -0.5. These are roughly the Canadian values before and after 1973. So, equation 3 is clearly the incorrect model for stochastic trend under this data generation scheme. The rest of the values are same as before. Again, for each simulation three models were estimated – the univariate and incorrect 'No Break' model using equation 3, the univariate and correct 'Break' model using equation 6 and the bivariate correct model using equation 6 instead of equation 3.

[3] Indeed, if the sample size is increased to 1000, the gain from the bivariate model is much smaller.

The results are in Table 2. In Panel A with negative trend cycle correlation, the coefficient of the standard deviation of the trend shock and the cycle shock is upward biased in the ‘No Break’ model whereas downward biased in the ‘Break’ model. The correlation coefficient of the trend cycle shocks was more biased and imprecise in the ‘Break’ model, though in the opposite direction. This is surprising since the ‘Break’ model is the more correct of the two univariate models. Again, the distribution of the correlation coefficient had a tendency to converge to either -1 or 1.

As against the univariate models, the bivariate model shows accurate estimates of the above parameters. These results go through in Panel B and Panel C. In Panel B we have estimates with zero correlation of trend cycle shocks and in Panel C we have estimates with strongly positive correlation of trend cycle shocks. The direction and size of biases in the correlation coefficient change between the univariate models in Panel B and Panel C. The standard deviation of the trend shock is consistently upward biased in the ‘No Break’ models and downward biased in the ‘Break’ models. As before, the bivariate models provide accurate estimates.

As the second exercise, the equation 5 corresponding to the non-output part of inflation was allowed to have three structural breaks in the intercept term. The equation is re-specified as:

7. $$x_{2t} = \beta_0 + \beta_0^{'} D_{1t} + \beta_0^{''} D_{2t} + \beta_0^{'''} D_{3t} + \beta_1 y_{2,t-1} + \varepsilon_{2,t}$$

In equation 7, D_{1t} is a dummy variable that takes the value of one between 46 and 85 datapoints, otherwise zero. Similarly, D_{2t} is a dummy variable that takes the value of one between 86 and 125 data points and D_{3t} takes the value of one for the remaining sample. The numbers roughly correspond to break dates in 1972, 1982 and 1992 as found in Rapach and Wohar (2003). The value of $\beta_0^{'}$ was chosen as 1.5 implying a doubling of the inflation, $\beta_0^{''}$ was chosen as zero – the 1980s disinflation and $\beta_0^{'''}$ as -0.25.

The data was simulated using equations 1 – 2, 4, 6 – 7 under previous values with different values of the correlation of the trend cycle shocks as before. The univariate models, the 'No Break' and 'Break' models, were estimated along with the correct bivariate model using equations 1 – 2, 4, 6 – 7. The results are in Table 3. In Panel A, the results are from negative correlation of trend cycle shock data. They reflect the same properties as Panel A of Table 2 – with fairly large difference between the estimates from univariate models and accurate results from the bivariate model. The estimates of the size of the breaks in the inflation equation, not presented in the table for space considerations, were accurate too. Similar patterns of the estimates are present in Panel B and Panel C of Table 3. Overall, the bivariate models do provide more accurate results even in the presence of breaks, both in output drift and in inflation.

A relevant issue at this point is determining when the bivariate model will not yield a better result than the univariate model. A trivial answer to this question is when there are no common factors ($\phi = 0$). However, there are two additional cases of interest. One, an increase in the persistence of the inflationary process decreased the precision of the results. Secondly, the results are sensitive to the values of the trend - inflation shock correlation and the cycle - inflation shock correlation. If one assumes a high value of the trend - inflation shock correlation and the cycle - inflation shock correlation was assumed to close enough to the trend - cycle shock correlation, the results became less precise and some simulations in the Monte Carlo experiments showed instability in convergence. The results are not reported due to space considerations.

3. The Inflation-GDP Unobserved Components Model of Canada and Its Estimates

Based on the results of the previous section, this section sets up a bivariate unobserved components model for estimating cyclical fluctuations in Canada. In this model, as in Watson (1986), Clark (1987), MNZ among others, the log GDP at time t, y_t, is a sum of a stochastic trend component, T_t and a cyclical component c_t.

8. $$y_t = T_t + c_t$$

The trend is assumed to follow a random walk with constant drift but recent evidence on trend productivity growth (e.g. see Kahn and Rich (2004), Roberts (2001) for the U.S. evidence and Kichian (1999) for Canada) show that a constant drift term may not be appropriate. Kichian (1999) and PW both use dummy variables to capture a drift term with breaks in the stochastic trend. Allowing the drift term to have multiple breaks within the sample period, the appropriate number of breaks in the drift is determined by Bai and Perron (1998, 2001, 2003, hereafter BP) multiple structural break tests on GDP growth. The dummies, $D_{T,i}$ capture these break dates in the drift term μ_0.

9. $$T_t = \mu_0 + \sum_{i=1}^{B_i} \mu_i D_{T,i} + T_{t-1} + \varepsilon_{T,t}$$

The cyclical component of output follows an autoregressive process with zero mean.

10. $$\theta_c(L)c_t = \varepsilon_{c,t}$$

Following Watson (1986), Harvey and Jaeger (1993) and MNZ, an autoregressive process of order two is assumed.

The second observable variable, inflation rate (π_t), depends on expected inflation, supply shocks (s_t) and the cyclical output. The expected inflation term can be modeled in several ways. One way is to simply assume lagged inflation as the expected inflation[4]. Another way is to model it in levels with appropriate number of lags and MA terms. The advantage of the second method is that the mean breaks in inflation (e.g. see Levin and Piger (2003), Rapach and Wohar (2005)) can be modeled as intercept shifts represented by dummies $(D_{\pi,j})$. The number of intercept breaks is to be determined by BP break tests.

11. $$\pi_t = \beta_{\pi,0,0} + \sum_{j=1}^{B_j} \beta_{\pi,0,j} D_{\pi,j} + \sum_{k=1}^{B_k} \beta_{\pi,1,k} \pi_{t-k} + \phi_c c_t + \sum_{m=1}^{B_m} \gamma_m s_{m,t} + \varphi_\pi (L) \varepsilon_{\pi,t}$$

Quarterly time series is obtained from International Financial Statistics maintained by the IMF, ranging from 1961:1 to 2004:4. The logarithm of real GDP, seasonally adjusted, times 100, is used as the output series. The inflation rate is based on seasonally adjusted CPI. The two supply shocks used here are the (natural logs of) demeaned nominal effective exchange rate change and the demeaned world average crude price change.

The empirics start out by performing a series of BP multiple structural break tests on inflation level and GDP growth[5]. The results are in Table 4. The UDMax and WDMax tests do suggest the existence of at least one break for the inflation rate. The sequential SupF tests imply three breaks in the mean. The result of three mean breaks and the corresponding estimates of the break dates confirm the findings reported by Demers (2003), Rapach and Wohar (2005)[6]. Interestingly, the third break date exactly

---

[4] Gerlach and Smets (1997) used first lag of inflation as the expected inflation in Canadian data – specifying the inflation equation in first differences. Kichian (1999) experimented with both first differenced inflation data and levels with mean breaks. She found the inflation level data with mean breaks to be a better fit. This paper found very similar results but does not report them here.

[5] See BP and Rapach and Wohar (2005) for a complete description of all the test statistics. The tests were performed on just the mean without including any lags. Maximum five breaks were allowed for each series with a minimum of 26 data points between each segments based on a 15 percent trimming value.

[6] Rapach and Wohar (2005) found four breaks in Canadian inflation. This study did not find the break they reported on 1966:1. However, the remaining three break dates are almost identical.

corresponds to Bank of Canada's announcement of inflation targeting in February, 1991. The tests for GDP growth show one break in the fourth quarter of 1973, the beginning of much researched productivity slowdown both in US and Canada. The estimated breaks dates along with the inflation and GDP growth date are in Figure 1.

Three dummy variables for the inflation equation (eqn. 11) and one dummy variable for the break in the drift term of GDP stochastic trend (eqn. 9) are generated. The parameters are estimated using maximum likelihood method and then the Kalman filter is used to get the estimates of the unobserved state variables. The number of own lags and the MA lags in the inflation equation is chosen according to the significance of the last lag. One lag of inflation and one MA lag proved to be adequate.

The parameter estimates for univariate correlated UC models, useful for comparison purposes, are in Table 5. The MNZ stochastic trend (eqn. 9 without break) is highly volatile but the PW (eqn. 9 with break) trend is almost non-stochastic after the break. These are very much in accordance with the US results. However, the MNZ cycle proved to be large for Canada (much larger than the Gerlach and Smets (1997) and Kichian (1999) estimates), although with fairly low persistence. The PW cycle is large and persistent. Figure 2 illustrates these results. The trend cycle shock correlation estimates yield sharply divergent results – almost perfectly negatively correlated to perfectly positively correlated[7]. So, the MNZ and PW methods provide very different results regarding the volatility of stochastic trend, persistence of cycle and trend-cycle correlation.

The results from bivariate model, eqns. 8 – 11 with break and correlated shocks, are in Table 6. The first result is that the estimate of standard deviation of the trend shock

[7] PW also found perfectly positively correlated trend-cycle shock in their unrestricted univariate model.

is fairly large and precise even after accounting for drift break. This is in contrast to the PW result that appropriately accounted drift breaks in a correlated trend cycle model produces non-stochastic trend. However, the point estimate of the standard deviation of the stochastic trend shock is lower than the MNZ estimates. The estimate of the standard deviation of the cyclical shock is moderate, lower than both MNZ and PW, but mildly imprecise. The cycle shows a fairly persistent movement; the sum of the autoregressive coefficients is 0.89. It is more persistent than MNZ cycle but less persistent than PW.

The trend-cycle correlation is highly negative and significant – thereby confirming the MNZ results. The rest of the correlations of the shocks are very close to zero and imprecise. The lagged inflation coefficient in the Phillips curve indicates a stationary process. The output gap coefficient in the Phillips curve is low but significantly different from zero. All the above three factors confirm the role of inflation in estimating trend and cycle from GDP data. The break in the drift term is negative and significant, confirming the PW estimates and the general observation of a slowdown in Canadian productivity from the mid-Seventies.

In Figure 3, the two – sided estimates of the stochastic trend and the cyclical fluctuations are shown. The trend is volatile before and after 1973. The shaded quarters are Economic Cycle Research Institute (ECRI) peak to trough movement in Canadian economic activity[8]. The cycle estimates nicely pick up all the three recessions. This illustrates the finding that bivariate estimation of output and inflation generates persistent economic cycles that describes the Canadian cycles quite well even after allowing for trend-cycle correlation, a stochastic trend and a drift break. One does not need to impose

[8] The February 2005 version of ECRI Canadian recession dates have only two recessions, 1981-82 and 1991-92. However, Bodman and Crosby (2000) reports three recessions from ECRI that also included 1974-75. All three are included in this paper.

a priori smoothness on the trend or zero correlation of trend cycle shocks when the trend is volatile. This confirms Kuttner's (1994) (also used by Gerlach and Smets (1997) and Kichian (1999) in the Canadian context) findings that inflation can improve the estimates of cyclical fluctuations.

For further comparison of the estimates to univariate detrending of GDP, a cubic time trend and a HP trend are also used. In Table 7, the standard deviations of the different cycle estimates and their correlations are shown. The graphs of the four univariate cycle estimates along with the bivariate cycle are in Figure 4. The MNZ cycle is the largest – though not persistent as one can see from the bottom left panel of Figure 4. The PW cycle is almost similar to time trend cycle and the HP cycle is the smallest. The bivariate cycle is larger than the HP cycle but smaller than the rest. To sum up, the bivariate model delivers a moderately large and persistent cycle that picks up the Canadian recessions quite well after accounting for all the shock correlations and breaks.

MNZ also argued that the estimates of the cycle are sensitive to the trend cycle correlation. This was re-examined in the above bivariate context by setting correlations of the inflation shocks and rest two shocks to zero. The bivariate model was then re-estimated six times; twice with zero trend cycle correlation – with and without allowing for the drift break and four more times with two perturbations around the zero correlation of trend cycle shocks. In two of them the correlation was set to -0.25 and in the other two the correlation was set to 0.25. The estimated cycles with restricted correlation along with the estimates of the unrestricted bivariate cycle (dashed lines) are in Figure 5.

The top-left panel in Figure 5 compares the zero correlation cycle with the unrestricted correlation cycle after allowing for the break. Both estimates display moderate persistence and pick up the Canadian recessions nicely. However, the magnitude of the fluctuations in the unrestricted correlation cycle is larger. The top-right panel compares the zero correlation cycle without break with the unrestricted correlation cycle. The results are quite similar to the top-left panel. The same exercise with -0.25 trend cycle shock correlation is illustrated in the middle-left and middle-right panels. The results again are very similar to the top panels. The bottom panels use 0.25 as the trend-cycle correlation value. The estimates of the restricted correlation cycle still pick up the recessions although the magnitudes of fluctuations are moderately different between the left and the right panel. Overall, allowing for different values of trend-cycle correlation does make the cycle estimates moderately sensitive – but they preserve the basic properties of persistence and recession dynamics quite accurately.

4. Using Random Walk Drift in the Bivariate Unobserved Components Model and Its Estimates

A series of recent studies, e.g. by Rao, Tang and Wang (2003), Wilson (2003), suggest that Canadian productivity growth accelerated from the second half of 1990s. The one break in the drift specification is not an adequate specification to capture this richer dynamics in the drift. Moreover, ignoring potential variations in the drift term may also bias the estimates of trend and cycle parameters. This section relaxes the restrictive assumption of just one break in the drift term by letting the drift term be a simple random walk.

The previous stochastic trend equation (eqn. 9) is replaced by the following stochastic trend equation:

12. $T_t = d_{t-1} + T_{t-1} + \varepsilon_{T,t}$

Where the drift term d_{t-1} follows a simple random walk:

13. $d_t = d_{t-1} + \varepsilon_{d,t}$

However, estimation of eqns. 8, 10 - 13 results in the well known 'pile-up[9]' problem of variance of the drift shock – even when it is uncorrelated with all other shocks. To avoid the 'pile-up' problem Stock and Watson's (1998) median unbiased estimation of the variance of the drift term is used.

The growth rate of stochastic trend estimates from the previous section is used to compute the exponential Wald statistics (Andrews and Ploberger (1994)) to test for one break in mean at an unknown break point. The computed Exp-Wald statistics is in Panel A of Table 8. Based on the Exp- Wald statistics, Table 3 of Stock and Watson (1998) is used to compute the median unbiased estimate of λ[10]. Equations 8 and 10 – 13 are then re-estimated using maximum likelihood with $\sigma_d = (\lambda / N) * \sigma_T$, where N is the sample size. The drift shock is also assumed to be uncorrelated with any other shock as the Stock and Watson (1998) method requires.

The results are in Panel B of Table 8. The point estimates are very similar to the ones in the previous section. The log likelihood value dropped due to two less parameters and also due to another diffused starting value for the random walk drift component. Addition of the random walk drift also resulted in a decline in the precision of the parameter estimates. The trend shock component is still large and the trend cycle shocks

[9] The problem is when the variance of the shock to a random walk component is small in an UC model, its maximum likelihood estimates are biased towards zero. See Stock (1994), Stock and Watson (1998) for discussions on this problem.

[10] The mean-Wald statistics also produced the same results.

are negatively correlated although the correlation coefficient is imprecisely estimated. The estimates of the cycle, presented in Figure 6, are persistent and picks up the three ECRI recessions nicely. It has a correlation of 0.67 with the estimates of the bivariate cycle in Section 3, also presented in Figure 6 for comparison purposes. In the graph the two measures of cycle have largely similar dynamics and recessionary features but they differ in the magnitude of the upswing before the recessions.

The estimate of standard deviation of the drift shock (σ_d) is 0.06, presented in Table 8[11], and the estimates of the drift component are presented in Figure 7. It depicts a productivity growth decline in Canada from early 1970s and a moderate recovery from the early 1990s. It confirms the productivity resurgence in Canadian economy during the 1990s as noted by Rao, Tang and Wang (2003) and Wilson (2003) – though the timing of turnaround is earlier than mid 1990s. Overall, this section does a robustness check on the previous results by introducing a random walk drift term in the stochastic trend. The point estimates confirm previous findings even with the random walk drift. The trend is still volatile suggesting most shocks are in the trend level and the cycle is moderate and persistent.

5. Conclusion

[11] The standard error of the estimate presented in the parenthesis was computed by conducting a Monte Carlo experiment, based on Roberts (2001). Data for stochastic trend was generated using the point estimates of standard deviation of the trend shock and the standard deviation of the drift shock. The sample size was 173, exactly the Canadian sample size. Then, the exponential – Wald test was conducted on growth rate of the trend and the corresponding median unbiased estimates of λ and σ_d were computed. This was repeated 5000 times. The number in the parenthesis is the standard deviation of the estimated σ_d s.

This paper presents three results on using the UC models to estimate business cycles, especially in the Canadian context. The Monte Carlo experiments show that the bivariate models deliver substantially more accurate results than the univariate models in a correlated UC framework with breaks in a standard post war sample. The stochastic trend for Canada is volatile even after drift breaks or random walk drifts. Estimates of the Canadian business cycle, estimated from inflation and output, are moderately large and persistent. The estimates also confirm the empirical result of negative trend-cycle correlation in the Canadian context.

References:

Andrews, D. W. K. and W. Ploberger (1994), "Optimal Tests When a Nuisance Parameter is Present Only Under the Alternative", *Econometrica*, 62, 1383-1414.

Apel, Mikael and P. Jansson (1999), "A Theory-consistent System Approach for Estimating Potential Output and the NAIRU", *Economics Letters*, 64 (3), 271-275.

Bai, Jushan and Pierre Perron (1998), "Estimating and Testing Linear Models with Multiple Structural Changes," *Econometrica*, 66, 47-68.

Bai, Jushan and Pierre Perron (2001), "Multiple Structural Change Models: A Simulation Analysis," forthcoming in *Econometric Essays*, D. Corbea, S. Durlauf and B.E. Hansen (eds.), Cambridge University Press.

Bai, Jushan and Pierre Perron (2003), "Computation and Analysis of Multiple Structural Change Models," *Journal of Applied Econometrics*, 18, 1-22.

Balke, Nathan and Mark Wohar (2002), "Low Frequency Movements in Stock Prices: A State-Space Decomposition," *Review of Economics and Statistics*, 84 (4), 649 -67.

Basistha, A. and C. Nelson (2005), "New Measures of Output Gap Based on the Forward Looking New Keynesian Phillips Curve", forthcoming in *Journal of Monetary Economics*.

Bodman, P. M. and M. Crosby (2000), "Phases of Canadian Business Cycle", *Canadian Journal of Economics*, 33 (3), 618-33.

Beveridge, S. and C. Nelson (1981), "A New Approach to the Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to the Measurement of the Business Cycle", *Journal of Monetary Economics*, 7, 151-174.

Clark, Peter K. (1987), “The Cyclical Component of US Economic Activity”, *Quarterly Journal of Economics*, 102, 797-814.

Demers, Frederick (2003), “The Canadian Phillips Curve and Regime Shifting”, Working Paper # 2003-32, *Bank of Canada*.

Gerlach, S. and F. Smets (1997), “Output Gaps and Inflation. Unobservable-components Estimates for the G-7 Countries”, Unpublished Paper, *Bank for International Settlements*, Basel.

Harvey, A. C. (1985), “Trends and Cycles in Macroeconomic Time Series”, *Journal of Business and Economic Statistic*s, 3, 216-27.

Harvey, A. C. and A. Jaeger (1993), “Detrending, Stylized Facts and the Business Cycle,” *Journal of Applied Econometrics*, 8 (3) 231-47.

Hodrick, R. J., and E. C. Prescott (1997), “Postwar U.S. Business Cycles: An Empirical Investigation”, *Journal of Money, Credit, and Bankin*g, 29 (1), 1-16.

Kahn, James and Robert Rich (2004), “Tracking the New Economy: Using Growth Theory to Detect Changes in Trend Productivity” Federal Reserve Bank of New York.

Kichian, Maral (1999), “Measuring Potential Output within a State-Space Framework”, Working Paper # 99-9, *Bank of Canada.*

Kuttner, Kenneth N. (1994); “Estimating Potential Output as a Latent Variable”, *Journal of Business and Economic Statistics*, 12, 361-368.

Levin, Andrew and Jeremy Piger (2003), “Is Inflation Persistence Intrinsic in Industrial Economies?” *Working Paper # 2002-023E*, Federal Reserve Bank of St. Louis.

Morley, J., Nelson, C. and E. Zivot (2003), "Why Are Beveridge-Nelson and Unobserved-Component Decompositions of GDP So Different?" *Review of Economics and Statistics*, 85, 235-243.

Oh, Kum Hwa and Eric Zivot (2006), "The Clark Model with Correlated Components", Working paper # 2006-06, *University of Washington*.

Perron, Pierre and Tatsuma Wada (2005), "Trends and Cycles: A New Approach and Explanations of Some Old Puzzles", working paper, Boston University.

Rapach, D. and Mark Wohar (2005), "Regime Changes in International Real Interest Rates: Are They a Monetary Phenomenon?" *Journal of Money, Credit, and Banking*, 37(5), 887-906.

Rao, Someshwar, J. Tang and W. Wang (2003), "Canada's Recent Productivity Record and Capital Accumulation", *International Productivity Monitor*, 7, Fall 2003.

Roberts, John (2001), "Estimates of the Productivity Trend Using Time-Varying Parameter Techniques", *Contributions to Macroeconomics*: *B. E. Journals in Macroeconomics*, 1 (1), article 3.

Schleicher, C. (2003), "Structural Time Series Models with Common Trends and Common Cycles", University of British Columbia, manuscript.

Sinclair, Tara (2005), "Permanent and Transitory Component in Output and Unemployment: Okun's Law Persists", Washington University, manuscript.

St-Amant, P. and S. van Norden (1997), "Measurement of the Output Gap: A Discussion of Recent Research at the Bank of Canada", Technical Report No. 79, *Bank of Canada*.

Stock, James H. (1994), "Unit Roots, Structural Breaks and Trends," *Handbook of Econometrics*, vol. 4, 2739 – 2841.

Stock, J. H. and M. W. Watson (1998), "Median Unbiased Estimation of Coefficient Variance in a Time-Varying Parameter Model," *Journal of American Statistical Association*, 93, 349-58.

Watson, M. W. (1986), "Univariate Detrending Methods with Stochastic Trends," *Journal of Monetary Economics*, 18, 49-75.

Wilson, Thomas (2003), "A Perspective on Future Productivity Growth in Canada", *International Productivity Monitor*, 7, Fall 2003.

Table 1: The Monte Carlo Simulation Results with No Break in the Output Drift

	Univariate results					*Bivariate results*			
Panel A: $\sigma_1 = 0.90, \sigma_2 = 1.0, \sigma_z = 0.71, \rho_{12} = 0, \rho_{1,z} = -0.79, \rho_{2z} = 0, \mu = 0.8$									
No Break	σ_1	σ_z	$\rho_{1,z}$	μ		σ_1	σ_2	σ_z	μ
	0.72 (0.30)	0.64 (0.17)	-0.35 (0.69)	0.80 (0.07)		0.89 (0.12)	0.99 (0.07)	0.70 (0.15)	0.80 (0.06)
Break	σ_1	σ_z	$\rho_{1,z}$	μ	μ_1	$\rho_{1,2}$	$\rho_{1,z}$	$\rho_{2,z}$	
	0.63 (0.32)	0.62 (0.16)	-0.12 (0.79)	0.80 (0.13)	0.00 (0.16)	-0.02 (0.15)	-0.77 (0.14)	0.00 (0.19)	
Panel B: $\sigma_1 = 0.90, \sigma_2 = 1.0, \sigma_z = 0.71, \rho_{12} = 0, \rho_{1,z} = 0, \rho_{2z} = 0, \mu = 0.8$									
No Break	σ_1	σ_z	$\rho_{1,z}$	μ		σ_1	σ_2	σ_z	μ
	0.76 (0.31)	0.71 (0.24)	0.39 (0.64)	0.80 (0.06)		0.87 (0.14)	0.99 (0.07)	0.69 (0.15)	0.80 (0.06)
Break	σ_1	σ_z	$\rho_{1,z}$	μ	μ_1	$\rho_{1,2}$	$\rho_{1,z}$	$\rho_{2,z}$	
	0.67 (0.33)	0.72 (0.25)	0.55 (0.60)	0.80 (0.14)	0.00 (0.16)	-0.02 (0.20)	0.07 (0.30)	-0.01 (0.25)	
Panel C: $\sigma_1 = 0.90, \sigma_2 = 1.0, \sigma_z = 0.71, \rho_{12} = 0, \rho_{1,z} = 0.79, \rho_{2z} = 0, \mu = 0.8$									
No Break	σ_1	σ_z	$\rho_{1,z}$	μ		σ_1	σ_2	σ_z	μ
	0.82 (0.33)	0.82 (0.30)	0.71 (0.48)	0.80 (0.07)		0.88 (0.13)	1.00 (0.06)	0.71 (0.13)	0.80 (0.06)
Break	σ_1	σ_z	$\rho_{1,z}$	μ	μ_1	$\rho_{1,2}$	$\rho_{1,z}$	$\rho_{2,z}$	
	0.73 (0.37)	0.86 (0.33)	0.80 (0.42)	0.80 (0.14)	0.01 (0.17)	0.02 (0.17)	0.80 (0.17)	-0.04 (0.20)	

Note: The standard deviations from the Monte Carlo simulations are in the parentheses. The sample size for each Monte Carlo simulation is 200 and the number of Monte Carlo simulations is 3000. The ‘No Break’ model is equations 1, 3 and 4 in the text. The ‘Break’ model is equations 1, 6 and 4 in the text. The ‘Bivariate’ model, which is also the data generating process, is equations 1 – 5 in the text.

Table 2: The Monte Carlo Simulation Results with One Break in the Output Drift

	Univariate results					*Bivariate results*			
Panel A: $\sigma_1 = 0.90, \sigma_2 = 1.0, \sigma_z = 0.71, \rho_{12} = 0, \rho_{1,z} = -0.79, \rho_{2z} = 0, \mu = 1.2, \mu_1 = -0.5$									
No Break	σ_1	σ_z	$\rho_{1,z}$	μ		σ_1	σ_2	σ_z	μ
	1.14 (0.25)	0.89 (0.27)	-0.91 (0.22)	0.82 (0.06)		0.89 (0.13)	0.99 (0.07)	0.70 (0.15)	1.20 (0.11)
Break	σ_1	σ_z	$\rho_{1,z}$	μ	μ_1	$\rho_{1,2}$	$\rho_{1,z}$	$\rho_{2,z}$	μ_1
	0.63 (0.32)	0.62 (0.16)	-0.12 (0.78)	1.20 (0.13)	-0.50 (0.16)	-0.02 (0.15)	-0.76 (0.15)	-0.00 (0.20)	-0.50 (0.12)
Panel B: $\sigma_1 = 0.90, \sigma_2 = 1.0, \sigma_z = 0.71, \rho_{12} = 0, \rho_{1,z} = 0, \rho_{2z} = 0, \mu = 1.2, \mu_1 = -0.5$									
No Break	σ_1	σ_z	$\rho_{1,z}$	μ		σ_1	σ_2	σ_z	μ
	1.23 (0.27)	0.87 (0.30)	-0.50 (0.47)	0.82 (0.07)		0.84 (0.16)	1.00 (0.07)	0.70 (0.16)	1.20 (0.13)
Break	σ_1	σ_z	$\rho_{1,z}$	μ	μ_1	$\rho_{1,2}$	$\rho_{1,z}$	$\rho_{2,z}$	μ_1
	0.67 (0.34)	0.73 (0.25)	0.52 (0.62)	1.20 (0.14)	-0.50 (0.16)	-0.04 (0.21)	0.11 (0.33)	0.01 (0.24)	-0.50 (0.15)
Panel C: $\sigma_1 = 0.90, \sigma_2 = 1.0, \sigma_z = 0.71, \rho_{12} = 0, \rho_{1,z} = 0.79, \rho_{2z} = 0, \mu = 1.2, \mu_1 = -0.5$									
No Break	σ_1	σ_z	$\rho_{1,z}$	μ		σ_1	σ_2	σ_z	μ
	1.26 (0.29)	0.87 (0.33)	-0.05 (0.61)	0.82 (0.07)		0.86 (0.14)	1.00 (0.06)	0.72 (0.14)	1.20 (0.11)
Break	σ_1	σ_z	$\rho_{1,z}$	μ	μ_1	$\rho_{1,2}$	$\rho_{1,z}$	$\rho_{2,z}$	μ_1
	0.72 (0.38)	0.87 (0.34)	0.78 (0.45)	1.20 (0.14)	-0.50 (0.16)	0.04 (0.18)	0.82 (0.16)	-0.05 (0.19)	-0.50 (0.12)

Note: The standard deviations from the Monte Carlo simulations are in the parentheses. The sample size for each Monte Carlo simulation is 200 and the number of Monte Carlo simulations is 3000. The ‘No Break’ model is equations 1, 3 and 4 in the text. The ‘Break’ model is equations 1, 6 and 4 in the text. The ‘Bivariate’ model, which is also the data generating process, is equations 1 - 2, 4 – 6 in the text.

Table 3: The Monte Carlo Simulation Results with One Break in the Output Drift and Three Inflation Breaks

	Univariate results					*Bivariate results*			
Panel A: $\sigma_1 = 0.90, \sigma_2 = 1.0, \sigma_z = 0.71, \rho_{12} = 0, \rho_{1,z} = -0.79, \rho_{2z} = 0, \mu = 1.2, \mu_1 = -0.5$									
No Break	σ_1	σ_z	$\rho_{1,z}$	μ		σ_1	σ_2	σ_z	μ
	1.14 (0.25)	0.89 (0.28)	-0.91 (0.22)	0.82 (0.07)		0.89 (0.13)	0.99 (0.07)	0.69 (0.16)	1.20 (0.13)
Break	σ_1	σ_z	$\rho_{1,z}$	μ	μ_1	$\rho_{1,2}$	$\rho_{1,z}$	$\rho_{2,z}$	μ_1
	0.62 (0.32)	0.61 (0.16)	-0.11 (0.78)	1.20 (0.13)	-0.50 (0.16)	-0.01 (0.16)	-0.78 (0.16)	-0.04 (0.21)	-0.50 (0.15)
Panel B: $\sigma_1 = 0.90, \sigma_2 = 1.0, \sigma_z = 0.71, \rho_{12} = 0, \rho_{1,z} = 0, \rho_{2z} = 0, \mu = 1.2, \mu_1 = -0.5$									
No Break	σ_1	σ_z	$\rho_{1,z}$	μ		σ_1	σ_2	σ_z	μ
	1.22 (0.28)	0.87 (0.32)	-0.51 (0.47)	0.82 (0.06)		0.85 (0.18)	1.00 (0.07)	0.69 (0.18)	1.20 (0.13)
Break	σ_1	σ_z	$\rho_{1,z}$	μ	μ_1	$\rho_{1,2}$	$\rho_{1,z}$	$\rho_{2,z}$	μ_1
	0.67 (0.34)	0.73 (0.25)	0.51 (0.62)	1.20 (0.14)	-0.50 (0.16)	-0.01 (0.22)	0.11 (0.38)	-0.03 (0.25)	-0.50 (0.16)
Panel C: $\sigma_1 = 0.90, \sigma_2 = 1.0, \sigma_z = 0.71, \rho_{12} = 0, \rho_{1,z} = 0.79, \rho_{2z} = 0, \mu = 1.2, \mu_1 = -0.5$									
No Break	σ_1	σ_z	$\rho_{1,z}$	μ		σ_1	σ_2	σ_z	μ
	1.26 (0.29)	0.88 (0.34)	-0.05 (0.61)	0.82 (0.07)		0.86 (0.15)	1.01 (0.07)	0.72 (0.15)	1.20 (0.13)
Break	σ_1	σ_z	$\rho_{1,z}$	μ	μ_1	$\rho_{1,2}$	$\rho_{1,z}$	$\rho_{2,z}$	μ_1
	0.72 (0.37)	0.86 (0.33)	0.79 (0.44)	1.20 (0.14)	-0.50 (0.16)	0.10 (0.18)	0.83 (0.15)	-0.12 (0.19)	-0.50 (0.15)

Note: The standard deviations from the Monte Carlo simulations are in the parentheses. The sample size for each Monte Carlo simulation is 200 and the number of Monte Carlo simulations is 3000. The 'No Break' model is equations 1, 3 and 4 in the text. The 'Break' model is equations 1, 6 and 4 in the text. The 'Bivariate' model, which is also the data generating process, is equations 1 - 2, 4, 6 – 7 in the text.

Table 4: Bai-Perron Multiple Structural Breaks Tests for Canada

	UDMax	WDMax	SupF(1|0)	SupF(2|1)	SupF(3|2)	SupF(4|3)	SupF(5|4)
Inflation	54.93*	79.25*	20.25*	17.14*	39.49*	2.05	-
GDP Growth	12.43*	12.43*	12.43*	1.28	1.84	1.68	1.12
Break Dates:		Inflation				GDP Growth	
	1972:2	1982:3	1991:1			1973:4	

Note: The structural break tests follow Bai and Perron (1998, 2001 and 2003) procedures using 5 maximum breaks and minimum 26 data points between the breaks based on 15 percent trimming value. The critical value for WDMax at the 5 percent level is 9.91. The 5 percent critical values for the following tests are: UDMax - 8.88, SupF(1|0) – 8.58, SupF(2|1) – 10.13, SupF(3|2) – 11.14, SupF(4|3) – 11.83 and SupF(5|4) – 12.25. The asterisked numbers are significant at the 5 percent level.

Table 5: Maximum Likelihood Estimates from Univariate UC Models for Canada

	MNZ		PW	
Standard	σ_T	σ_c	σ_T	σ_c
Deviations	1.49 (0.35)	1.96 (0.50)	0.05 (0.48)	0.74 (0.47)
Autoregressive	$\theta_c(1)$	$\theta_c(2)$	$\theta_c(1)$	$\theta_c(2)$
Parameters	0.80 (0.25)	-0.04 (0.06)	1.22 (0.19)	-0.28 (0.19)
Drift and	μ_0		μ_0	μ_1
Break	0.88 (0.11)		1.28 (0.07)	-0.57 (0.09)
Correlation and	$\rho_{T,c}$	$LogL$	$\rho_{T,c}$	$LogL$
Likelihood	-0.99 (0.01)	-215.85	1.00 (0.00)	-211.07

Note: The numbers in the parentheses are standard errors computed using the delta method. Both 'MNZ' and 'PW' models are equations 8 – 10 in the text. However, the 'MNZ' model does not allow for a break in the drift in 1973:4, whereas 'PW' model does allow for that break.

Table 6: Maximum Likelihood Estimates from the Bivariate UC Model

The Standard Deviations and the Correlations of the Shocks					
σ_T	σ_c	σ_π	$\rho_{T,c}$	$\rho_{T,\pi}$	$\rho_{c,\pi}$
0.93 (0.18)	0.51 (0.30)	1.35 (0.18)	-0.77 (0.26)	-0.08 (0.12)	-0.05 (0.20)
The AR terms, the Drift terms and the Output Gap Coefficient in the Phillips Curve					
$\theta_c(1)$	$\theta_c(2)$	μ_0	μ_1	ϕ_c	
1.61 (0.19)	-0.72 (0.17)	1.24 (0.14)	-0.51 (0.16)	0.11 (0.05)	
The Other Phillips Curve Coefficients and the Log Likelihood					
$\beta_{\pi,1,1}$	$\varphi_\pi(1)$	γ_1	γ_2	$LogL$	
0.75 (0.04)	-1.23 (0.21)	-0.017 (0.012)	0.006 (0.002)	-389.95	

Note: The numbers in the parentheses are standard errors computed using delta method. The bivariate model is equations 8 – 11 in the text, with one drift break and three inflation breaks.

Table 7: Standard deviation and Correlation Matrix with Different Univariate Estimates

	Time	HP	MNZ	PW	Bivariate
Time	2.33				
HP	0.85	1.38			
MNZ	0.37	0.50	3.07		
PW	0.90	0.72	0.31	2.59	
Bivariate	0.53	0.69	0.43	0.55	1.64

Note: The diagonal terms are the standard deviations of the cycle estimates from the respective methods. The off-diagonal terms are the correlation coefficients of the respective cyclical measures.

Table 8: Parameter Estimates from the Bivariate Model with a Random Walk Drift

Panel A: The Exponential-Wald Test on Trend GDP Growth

Exp-Wald = 6.122, $\lambda / N = 0.077$

Panel B: Maximum Likelihood Estimates from the Bivariate UC Model With RW Drift

The Standard Deviations and the Correlations of the Shocks

σ_T	σ_c	σ_π	σ_d
0.80 (0.33)	0.45 (0.81)	1.52 (0.32)	0.06 (0.04)*
$\rho_{T,c}$	$\rho_{T,\pi}$	$\rho_{c,\pi}$	
-0.52 (0.88)	-0.03 (0.08)	-0.14 (0.47)	

The AR terms and the Output Gap Coefficient in the Phillips Curve

$\theta_c(1)$	$\theta_c(2)$	ϕ_c
1.59 (0.58)	-0.69 (0.50)	0.12 (0.05)

The Other Phillips Curve Coefficients and the Log Likelihood

$\beta_{\pi,1,1}$	$\varphi_\pi(1)$	γ_1	γ_2	$LogL$
0.76 (0.08)	-1.01 (0.37)	-0.018 (0.013)	0.005 (0.002)	-400.60

Note: In Panel B, except for the '*'-ed entry, the numbers in the parentheses are standard errors computed using the delta method. The estimated model is equations 8 and 10 – 13 in the text. The '*'-ed entry is the estimated standard deviation of the drift shock and the number in the parenthesis is the standard error based on 5000 Monte Carlo simulations.

Figure 1: Canadian Inflation and GDP Growth with the Break Dates

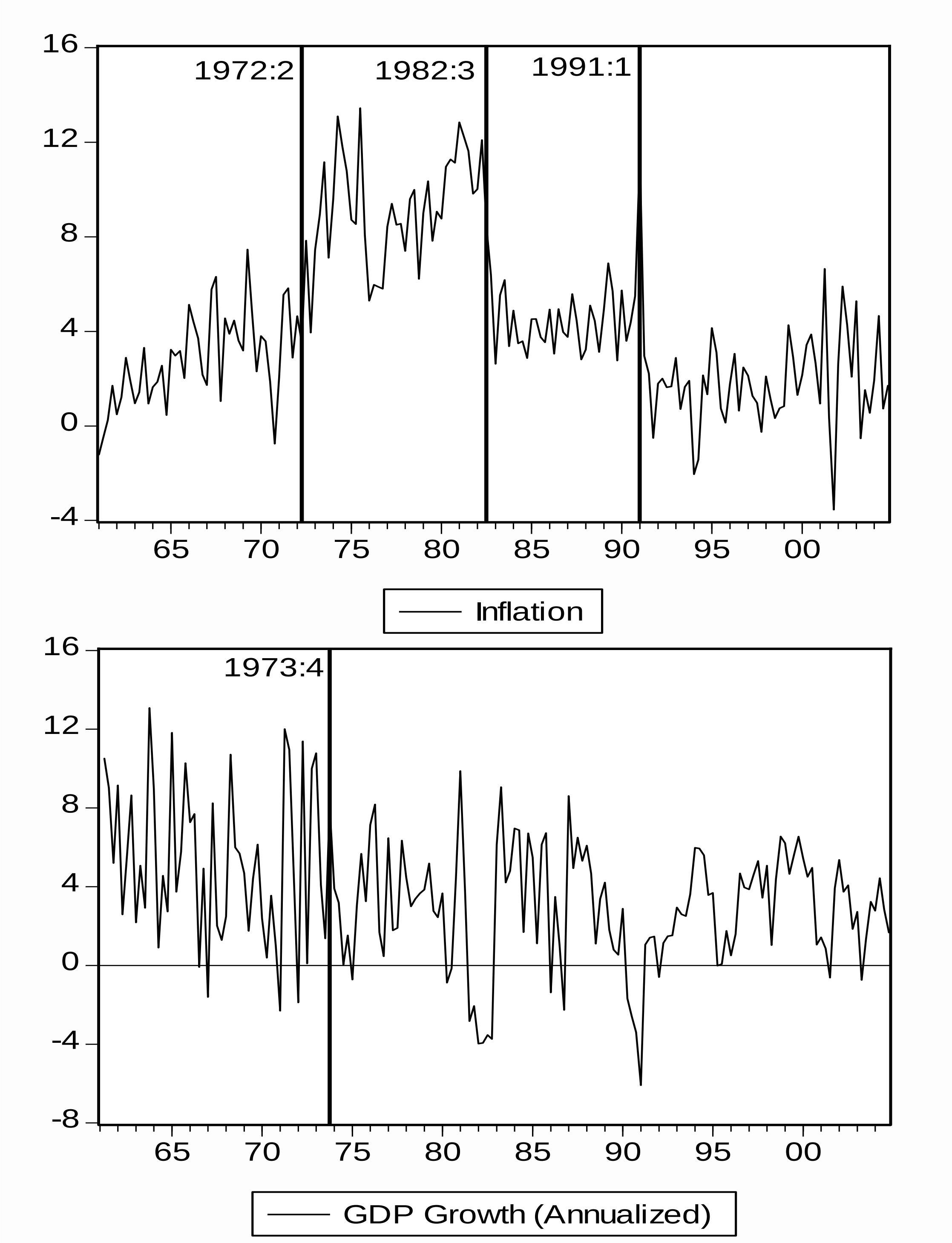
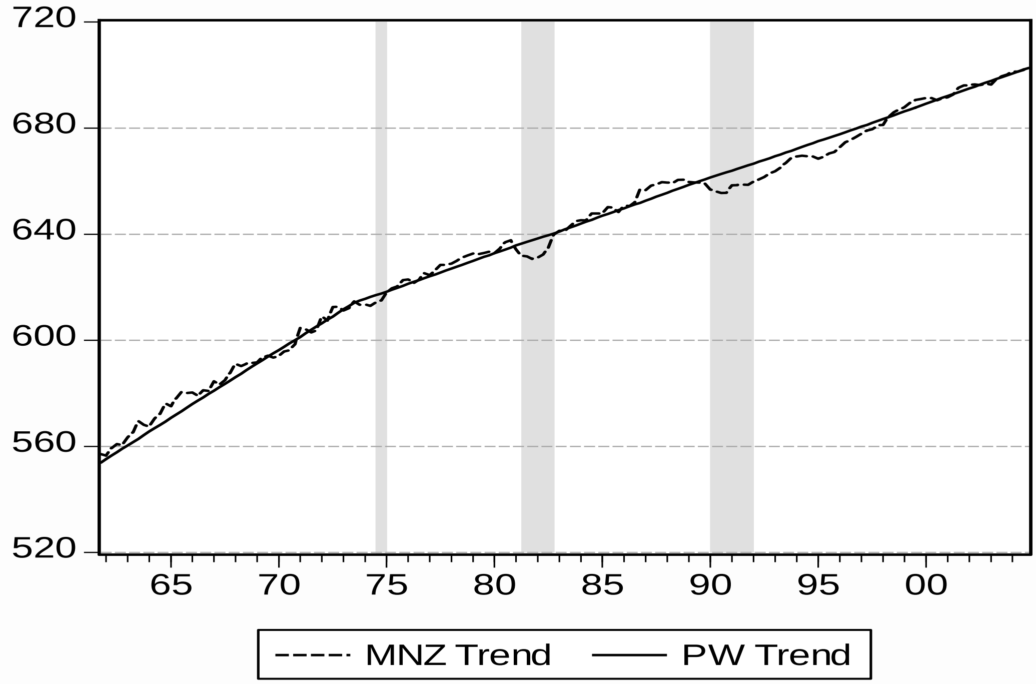


Figure 2: Estimates of Trend and Cyclical Fluctuations in Canada from the Univariate Models



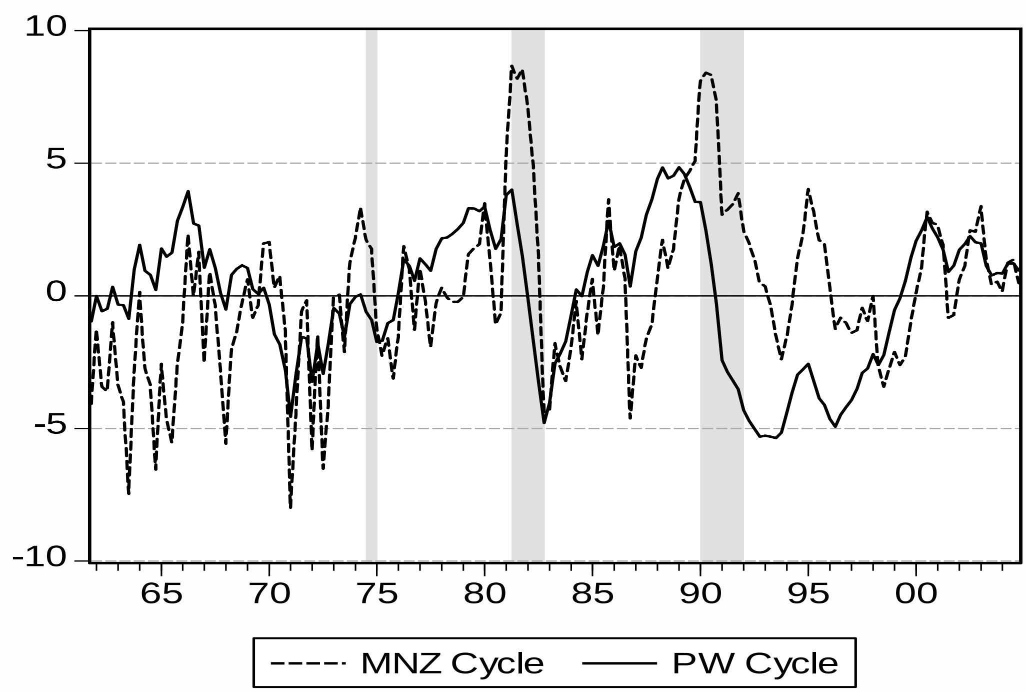
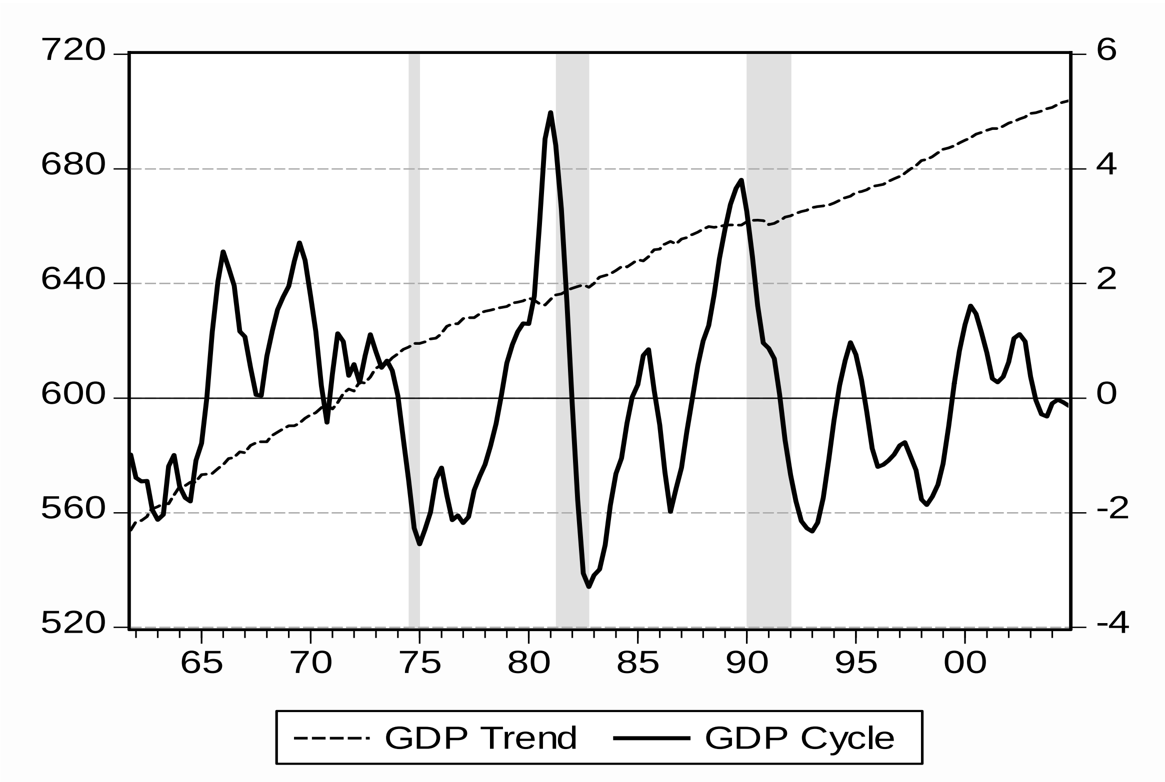
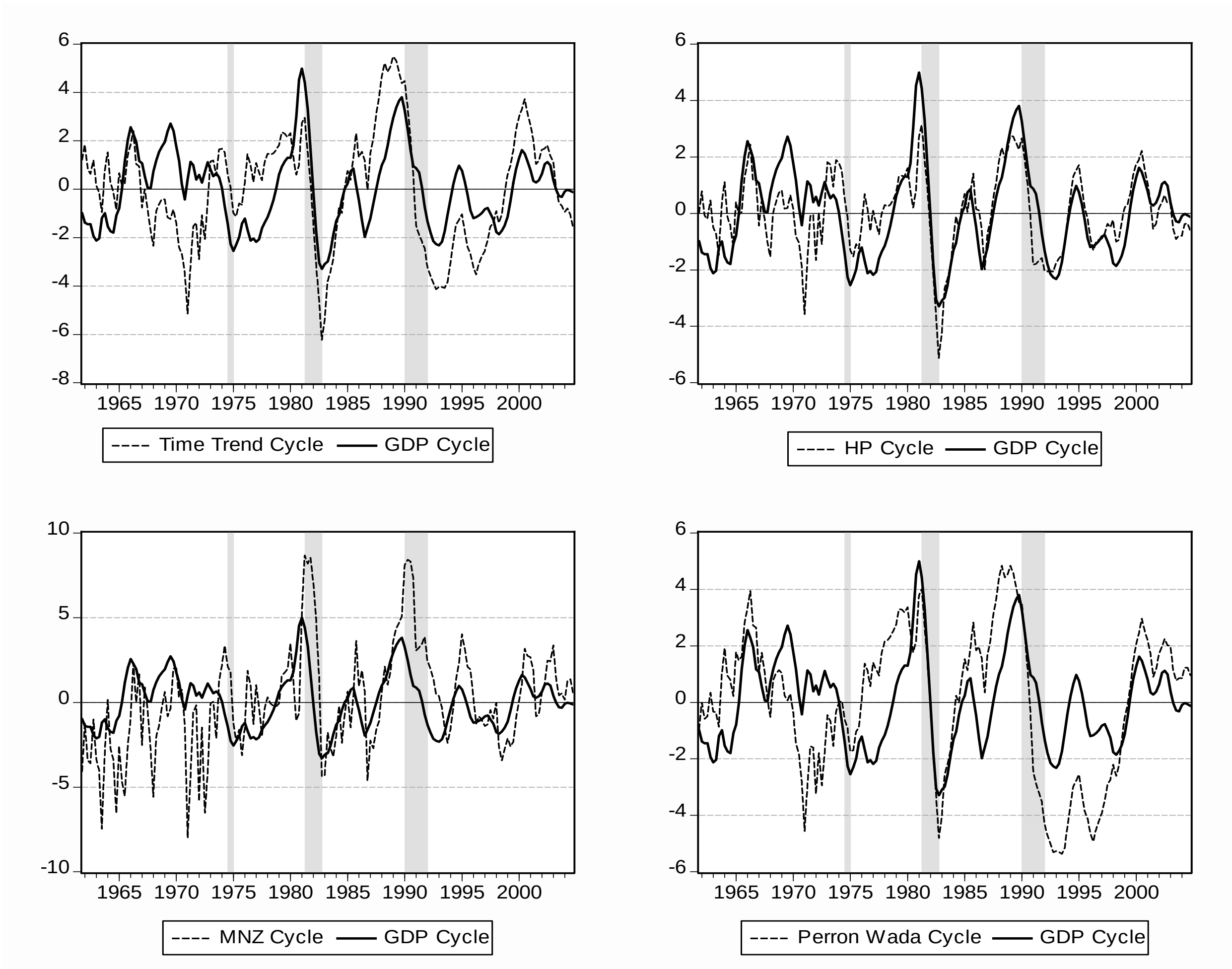


Figure 3: Estimates of Trend and Cyclical Fluctuations in Canada from the Bivariate Model



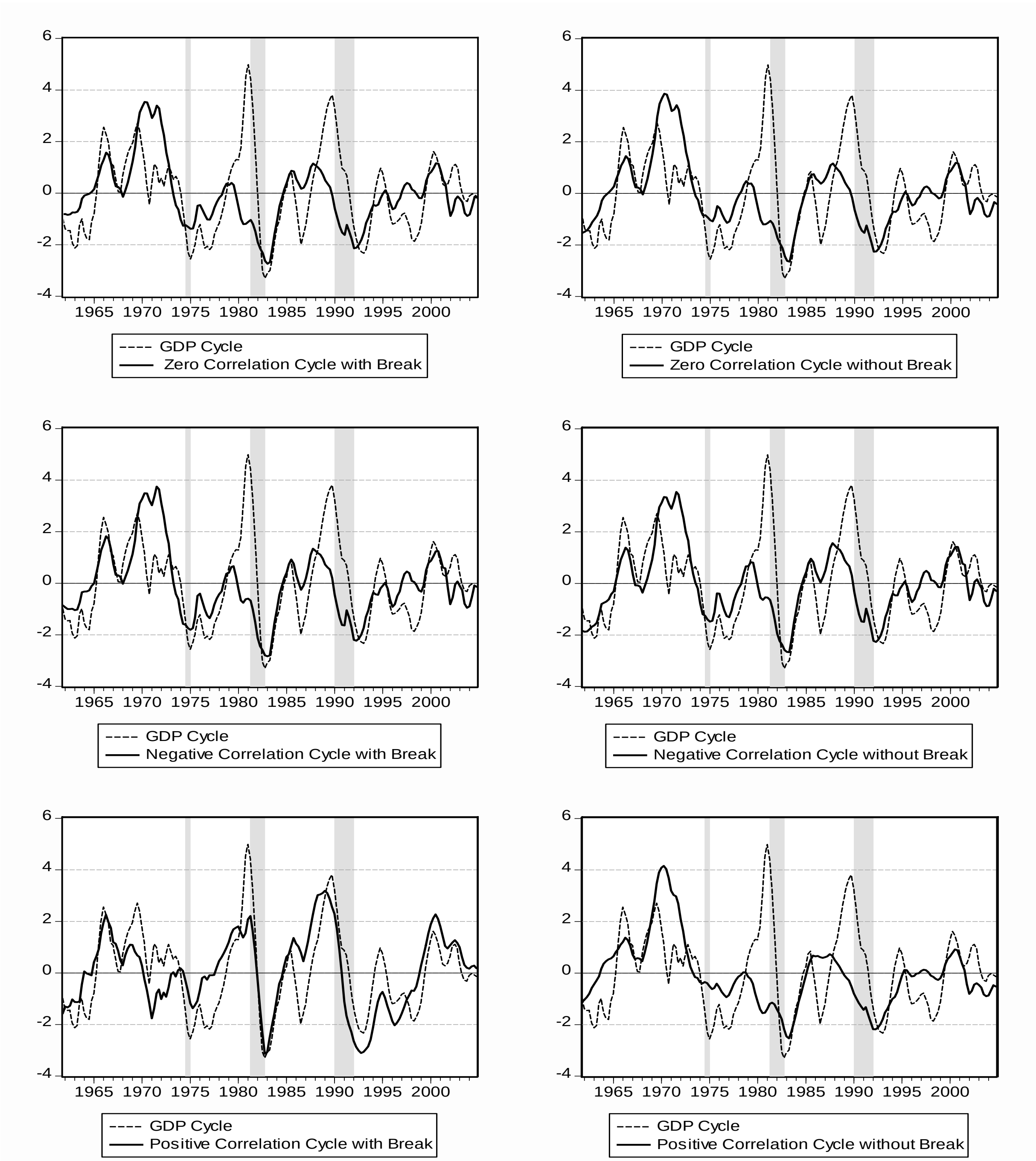
Note: The left scale is for trend output measured in natural logarithm times 100 and the right scale is for cycle measured in percentage points.

Figure 4: Comparison of Cyclical Fluctuations from the Correlated Bivariate Model with Univariate Methods



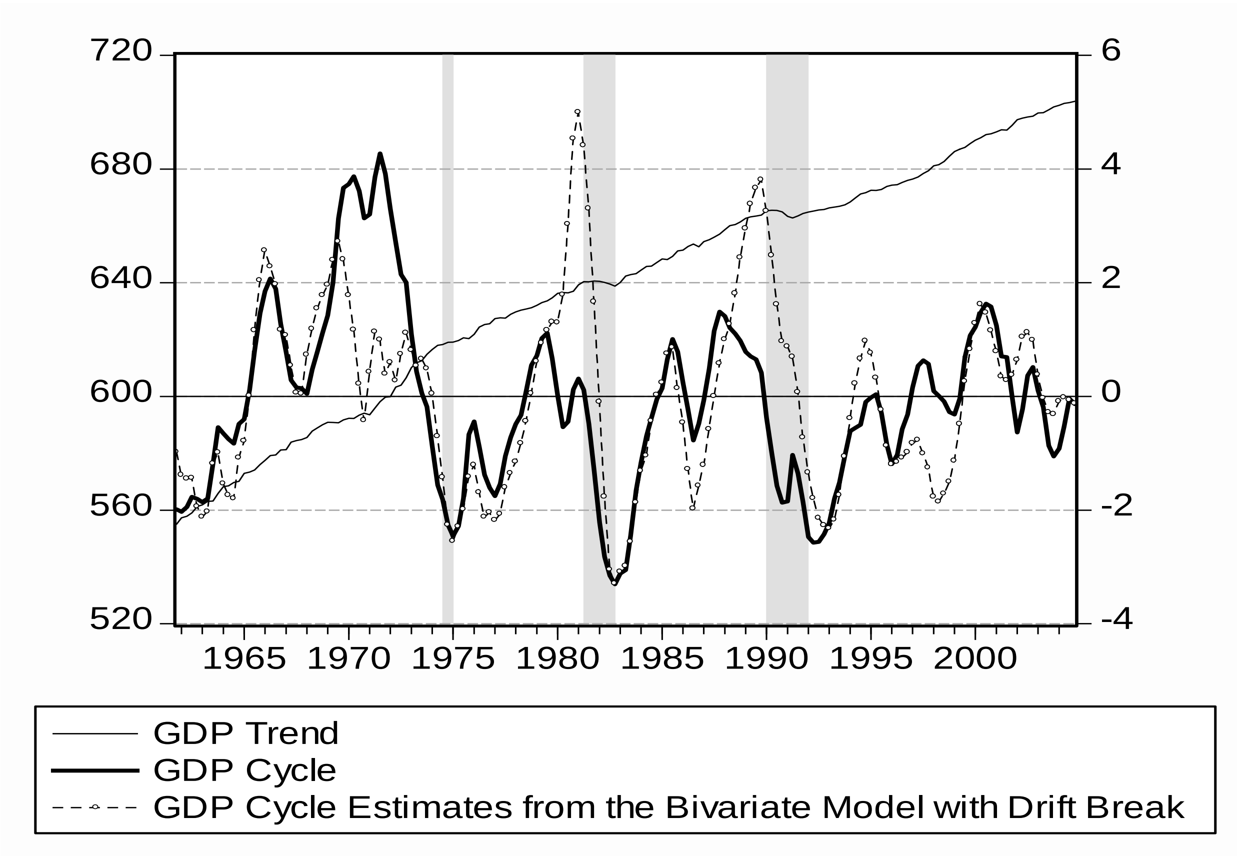
Note: Each panel compares the estimated cyclical component from the univariate models with the GDP cycle estimates from the correlated bivariate model. In each panel ‘GDP Cycle’ implies the GDP cycle estimates from Figure 3.

Figure 5: Comparison of Cyclical Fluctuations from the Correlated Bivariate Model with Uncorrelated Models



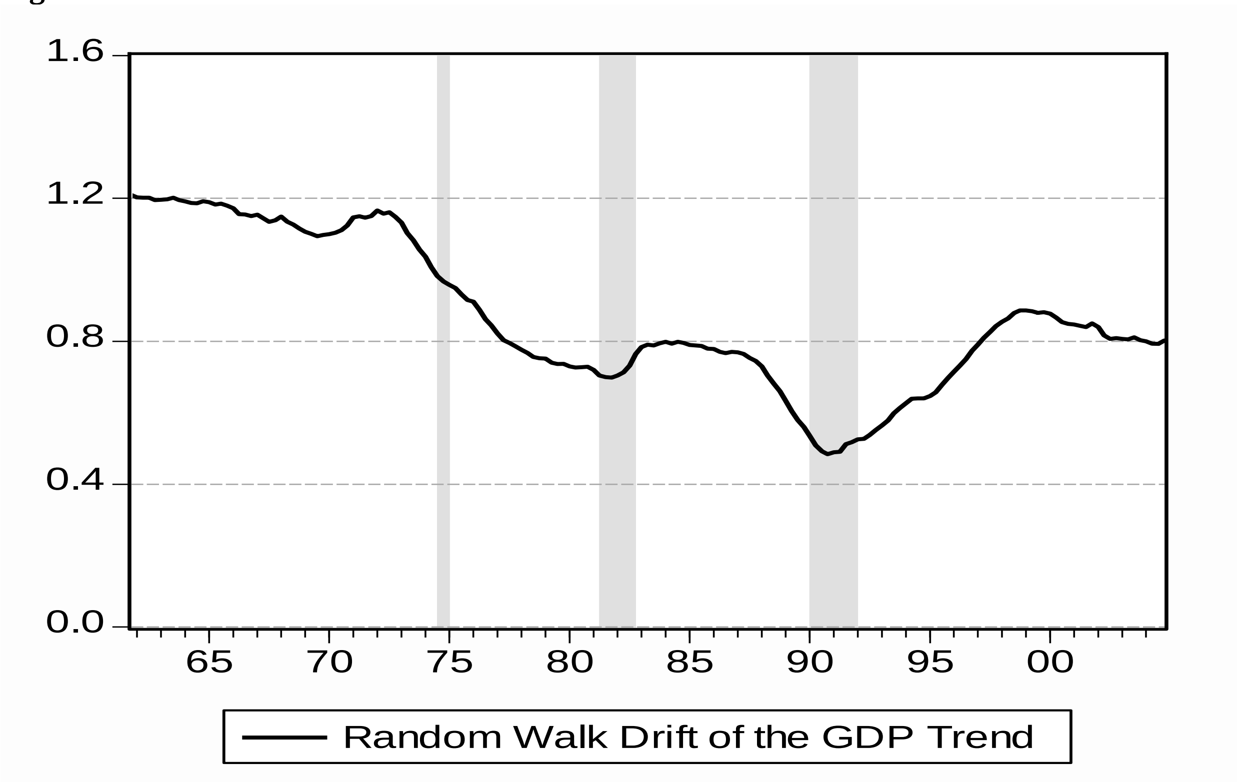
Note: Each panel compares the estimated cyclical component from the bivariate model under different assumptions about trend – cycle correlations; zero, negative (-0.25) and positive (0.25). The inflation shocks are assumed to be uncorrelated with other shocks. In each panel 'GDP Cycle' implies the GDP cycle estimates from Figure 3 shown here for comparison purposes. The left panels allow for a break in the drift term of GDP trend whereas the right panels do not.

Figure 6: Estimates of Trend and Cyclical Fluctuations in Canada from the Bivariate Model with the Random Walk Drift



Note: The left scale is for trend output measured in natural logarithm times 100 and the right scale is for cycle measured in percentage points. The “GDP Cycle Estimates from the Bivariate Model with Drift Break” shown here for comparison purposes is the “GDP Cycle” from Figure 3.

Figure 7: Estimates of the Random Walk Drift in Canada



Appendix: The Bivariate State-Space Model of the Canadian Economy

The section presents the state-space form of the bivariate model used for estimation in Section 3. The symbols have their usual meanings described in the text. The measurement equations in the compact form are:

Measurement Equations:

$$y_t = T_t + c_t$$

$$\pi_t = \tilde{\pi}_t + \phi_c c_t$$

The actual inflation equation described in the text was:

$$\pi_t = \beta_{\pi,0,0} + \sum_{j=1}^{B_j} \beta_{\pi,0,j} D_{\pi,j} + \sum_{k=1}^{B_k} \beta_{\pi,1,k} \pi_{t-k} + \phi_c c_t + \sum_{m=1}^{B_m} \gamma_m s_{m,t} + \varphi_\pi (L) \varepsilon_{\pi,t}$$

The term $\tilde{\pi}_t$ denotes the non-cyclical part of the inflation dynamics with three breaks, one lag of inflation and one MA lag. The drift term in the stochastic trend equation has one break and the cyclical gap term is AR (2).

The transition equations can be written in the vector form as:

Transition equations:

$$S_t = A_t + FS_{t-1} + \mathrm{E}_t$$

where

$$S_t = \begin{bmatrix} T_t \\ c_t \\ c_{t-1} \\ \tilde{\pi}_t \\ \varepsilon_{\pi,t} \end{bmatrix}, \quad A_t = \begin{bmatrix} \mu_0 + \mu_1 D_{T,1} \\ 0 \\ 0 \\ \beta_{\pi,0,0} + \beta_{\pi,0,1} D_{\pi,1} + \beta_{\pi,0,2} D_{\pi,2} + \beta_{\pi,0,3} D_{\pi,3} + \beta_{\pi,1,1} \pi_{t-1} \\ 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \theta_c(1) & \theta_c(2) & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \varphi_\pi(1) \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathrm{E}_t = \begin{bmatrix} \varepsilon_{T,t} \\ \varepsilon_{c,t} \\ 0 \\ \varepsilon_{\pi,t} \\ \varepsilon_{\pi,t} \end{bmatrix}$$

The variance covariance matrix of the shocks was restricted to be positive definite using Choleski decomposition. The AR(2) dynamics of the cyclical component was restricted to be stationary with possible imaginary roots (described in Oh and Zivot (2006)). The initial values of the state variables c_t , c_{t-1} and $\varepsilon_{\pi,t}$ were their steady state values of zero. For T_t start-up value, previous quarter's log GDP value was used with a high variance (to make it diffused) and for $\tilde{\pi}_t$ the three percent average value over the sample was used.