Estimating Earnings Trend Using Unobserved Components Framework

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Abstract

Regressions using valuation ratios for predicting long term stock returns often use a ten-year moving average of earnings as a proxy for unobserved future earnings. This paper shows that the earnings trend can be directly estimated using bivariate unobserved components models. The results show that the valuation ratios based on the estimated trends improve the fit of stock return predictive regressions. However, the 90 percent confidence intervals around the estimated trends are large and tend to include the moving average trend.

JEL Classification: C22, C51, G12.

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Introduction

Empirical studies by Campbell and Shiller (1998, 2001) show that valuation ratios can be used for predicting long term real stock returns. In particular, they use the price-smoothed earnings ratio computed by assuming a ten-year moving average of real earnings as earnings 'trend,' i.e., the component that captures long term future earnings.[1] Computing a multi-year moving average reduces the effect of cyclical fluctuations on corporate earnings. Averaging past earnings in valuation analysis is a common approach first recommended by Graham and Dodd (1934), but it may not be the best way to model the earnings trend. This paper relaxes the assumption of the moving average trend by letting the data speak. We directly estimate the earnings trend using unobserved components models. Our models use both the real earnings data and long-term real stock returns data. The unobserved components framework also allows us to compare the performance of different statistical assumptions by using the Schwarz Information Criterion.

The theoretically and statistically appealing motivation for estimating the earnings trend is derived from the Beveridge-Nelson (1981) decomposition of a time series into its trend and cycle components. The trend can be interpreted as a long term conditional forecast of the time series. Morley (2002) and Morley, Nelson and Zivot (2003) show that the Beveridge-Nelson decomposition can be usefully cast into an unobserved components framework which allows the trend and the cycle shocks to be correlated. Further studies by Ord et al. (1998) and Anderson et al. (2006) show that the perfect correlation between the shocks, as in the Beveridge-Nelson decomposition, can be modeled as a single source shock. Laubach (2001) argues in a different context of estimating NAIRUs that bivariate modeling can also help to reduce the uncertainty around the estimated unobserved components.

[1] Many other empirical studies also use the price-smoothed earnings ratio for predicting stock returns. These studies are not mentioned due to space considerations.

This paper uses the above developments in the unobserved components modeling to estimate the earnings trend under three different assumptions about correlation of shocks. The results indicate a fairly volatile earnings trend in all three cases. The fit of the long-run stock returns predictive regression is higher in all the cases relative to the moving average trend. However, the 90 percent confidence intervals of the estimated trends tend to include the moving average trend.

The unobserved components model and the estimates

The following unobserved components model is used for estimating the earnings trend. The first measurement equation uses log real earnings, e_t, to be decomposed into two unobserved components: its permanent (or stochastic trend) part p_t and the cyclical part c_t.

$$e_t = p_t + c_t \tag{1}$$

The permanent part is assumed to follow a random walk with a constant drift μ in equation (2). The cyclical part c_t is assumed to follow an autoregressive process in equation (3). Following Morley (2001) and Morley et al. (2003), we use an autoregressive process of order two.

$$p_t = \mu + p_{t-1} + \varepsilon_t \tag{2}$$

$$\phi(L)c_t = \omega_t \tag{3}$$

The second measurement equation, equation (4), is the future stock returns equation. The stock return over the next ten years, r_{t+1}^{10}, depends on a constant, the valuation ratio shown by log real stock price (s_t) minus the permanent part of log real earnings (p_t) and a serially correlated unobserved component f_{t+1} to account for other omitted factors. The f_{t+1} component is assumed

to take a moving average form based on comparison of SICs (not reported) in equation (5). We use ten lags in the moving average process.

$$r_{t+1}^{10} = \alpha + \beta(s_t - p_t) + f_{t+1} \tag{4}$$

$$f_{t+1} = \theta(L)v_{t+1} \tag{5}$$

We use three specifications of the correlations between the three shocks ε_t, ω_t and v_t. In the first specification, they are assumed to be uncorrelated as in Clark (1987). In the second and third specifications, following Anderson et al. (2006) which uses the Beveridge-Nelson result of perfectly negative correlation of the shocks, ε_t and ω_t are assumed to be perfectly correlated and modeled as a single source shock using $\omega_t = \gamma\varepsilon_t$. The shock ε_t is assumed to be uncorrelated to v_{t+1} in the second specification and we allow for that correlation to be estimated in the third specification.

We use annual average data ranging from 1871 to 2007 obtained from Robert Shiller's web site. The ten-year stock return is computed from 1871 to 1997 where the 1997 stock return denotes the stock return from 1997 to 2007. Therefore, based on the data availability, we are able to estimate the earnings trend from 1871 to 1996. We estimate the parameters of the model using maximum likelihood and then use the Kalman filter to obtain the estimates of the trend. The standard errors of the estimated trends are computed using Hamilton's (1986) procedure and are based on 1000 Monte Carlo replications.

The results are reported in Table 1 and the estimated trends are shown in Figure 1. Panel A of Table 1 shows that the valuation ratio based on the ten-year moving average of earnings predicts future long term stock returns with a fit of 19 percent. The upper-left panel of Figure 1 shows the moving average trend. Panel B of Table 1 shows the parameter estimates from the

bivariate unobserved components model with uncorrelated shocks. The estimate of μ shows that real earning grew at an average annual rate of 1.4 percent. The standard deviations of all shocks are moderate and fairly precisely estimated. The estimated earnings trend is shown in the upper-right of Figure 1 along with its 90 percent confidence interval and the moving average trend. One can observe that although the estimated earnings trend appears to be fairly different than the moving average trend, the 90 percent confidence interval does include the moving average trend almost all of the time. The regression of the long-term future stock returns on the estimated earnings trend has a fit of about 29 percent, which is 10 percent higher than the moving average trend is used.

Panel C of Table 1 reports the parameter estimates using a single source of shock between the trend and cycle of real earnings but uncorrelated with shocks to future real stock returns ν_{t+1}. The estimates are largely similar to those in Panel B, although the point estimate of the standard deviation of the shock to the earnings trend is higher, implying a more volatile trend. The lower-left panel of Figure 1 shows the estimated trend, which appears to be more volatile than the trend in the uncorrelated case. The fit of the stock returns regression on the valuation ratio based on the estimated trend is still about 29 percent.

Panel D of Table 1 reports the parameter estimates using a single source of shock between the trend and cycle of real earnings and correlated with shocks to future real stock returns ν_{t+1}. The estimates are largely similar to those in Panel C. The point estimate of the standard deviation of the shock to earnings trend is higher than that reported in Panel B, implying a more volatile trend. The correlation between the trend shock and the future stock returns shock is estimated to be -0.23, low but precisely estimated. The lower-right panel of Figure 1 shows the estimated trend which appears to be more volatile than the trend in the

uncorrelated case. The fit of the stock returns regression on the valuation ratio based on the estimated trend is marginally lower at about 28 percent but still higher the 19 percent fit reported in Panel A. This model also shows the lowest SIC of the three unobserved components models, implying that it provides the best description for the data.

The $(s_t - p_t)$ term in equation (4) can be viewed (up to a constant) as a deviation of the aggregate stock prices from the earnings trend, or as a proxy for market mispricing. Brown and Cliff (2005) find that stock market valuation errors are positively correlated with investor sentiment. We examined the relation between our mispricing proxy and the investor sentiment index from Baker and Wurgler (2006). The sentiment index is estimated as the first principal component of the closed-end fund discount, equity share in new security issues, and lagged NYSE turnover. The coefficient of the sentiment index in a linear regression using the 1934-1996 period was positive and significant at the 10 percent level.[2]

Conclusion

The main contribution of this paper is to show that the earnings trend can be estimated by using a bivariate unobserved components framework under different assumptions about the correlations of the shocks. All three estimated trends show a better fit of the regression equation using the price-earnings ratio to predict long term stock returns than the traditional specification using the moving average earnings trend. The drawback of the estimated trends is that their 90 percent confidence interval almost always includes the moving average trend. However, this drawback will become less of an issue with availability of more data, making the unobserved components models an asymptotically attractive choice.

[2] These results are not tabulated to save space, but are available upon request.

References

Anderson, H. M., Low, C. N. and R. Snyder (2006); "Single source of error state space approach to the Beveridge Nelson decomposition", *Economics Letters*, 91, 104-109.

Baker, M. and J. Wurgler (2006); "Investor Sentiment and the Cross-Section of Stock Returns," *Journal of Finance*, 61, 1645-1680.

Beveridge, S. and C. Nelson (1981); "A New Approach to the Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to the Measurement of the Business Cycle", *Journal of Monetary Economics*, 7, 151-174.

Brown, G. W. and M. T. Cliff (2005); "Investor Sentiment and Asset Valuation," *Journal of Business*, 78, 405-440.

Campbell, J. Y. and R. J. Shiller (1998); "Valuation Ratios and the Long-Run Stock Market Outlook", *Journal of Portfolio Management*, 24, 11- 26.

Campbell, J. Y. and R. J. Shiller (2001); "Valuation Ratios and the Long-Run Stock Market Outlook: An Update", *NBER Working Paper Series*, no. 8221.

Clark, P. K. (1987); "The Cyclical Component of US Economic Activity", *Quarterly Journal of Economics*, 102, 797-814.

Graham, B. and D. L. Dodd (1934); "Security Analysis," First Edition. New York: McGraw Hill.

Hamilton, J. (1986), "A Standard Error for the Estimated State Vector of a State Space Model," *Journal of Econometrics*, 33, 387-397.

Laubach, T. (2001); "Measuring the NAIRU: Evidence from Seven Economies", *The Review of Economics and Statistics*, 83, 218-231.

Morley, J. (2002); "A state-space approach to calculating the Beveridge-Nelson decomposition", *Economics Letters*, 75, 123-127.

Morley, J., Nelson, C. and E. Zivot (2003); "Why Are Beveridge-Nelson and Unobserved-Component Decompositions of GDP So Different?", *Review of Economics and Statistics*, 85, 235-243.

Ord, J. K., Koehler, A. B. and R. Snyder (1997); "Estimation and Prediction for a class of dynamic nonlinear statistical models", *Journal of American Statistical Association*, 92, 1621-1629.

Table 1: Parameter Estimates from the Unobserved Components Models

Panel A: The 10 Year Moving Average Regression

α	β	R^2
1.987 (0.35)	-0.671 (0.13)	0.186

Panel B: The UC Model with Uncorrelated Shocks

μ	σ_ε	σ_ν	σ_ω	R^2	SIC
0.014 (0.01)	0.056 (0.02)	0.137 (0.01)	0.123 (0.01)	0.288	-1.158

Panel C: The UC Model with Single Shock for Earnings and Uncorrelated with Stock Returns

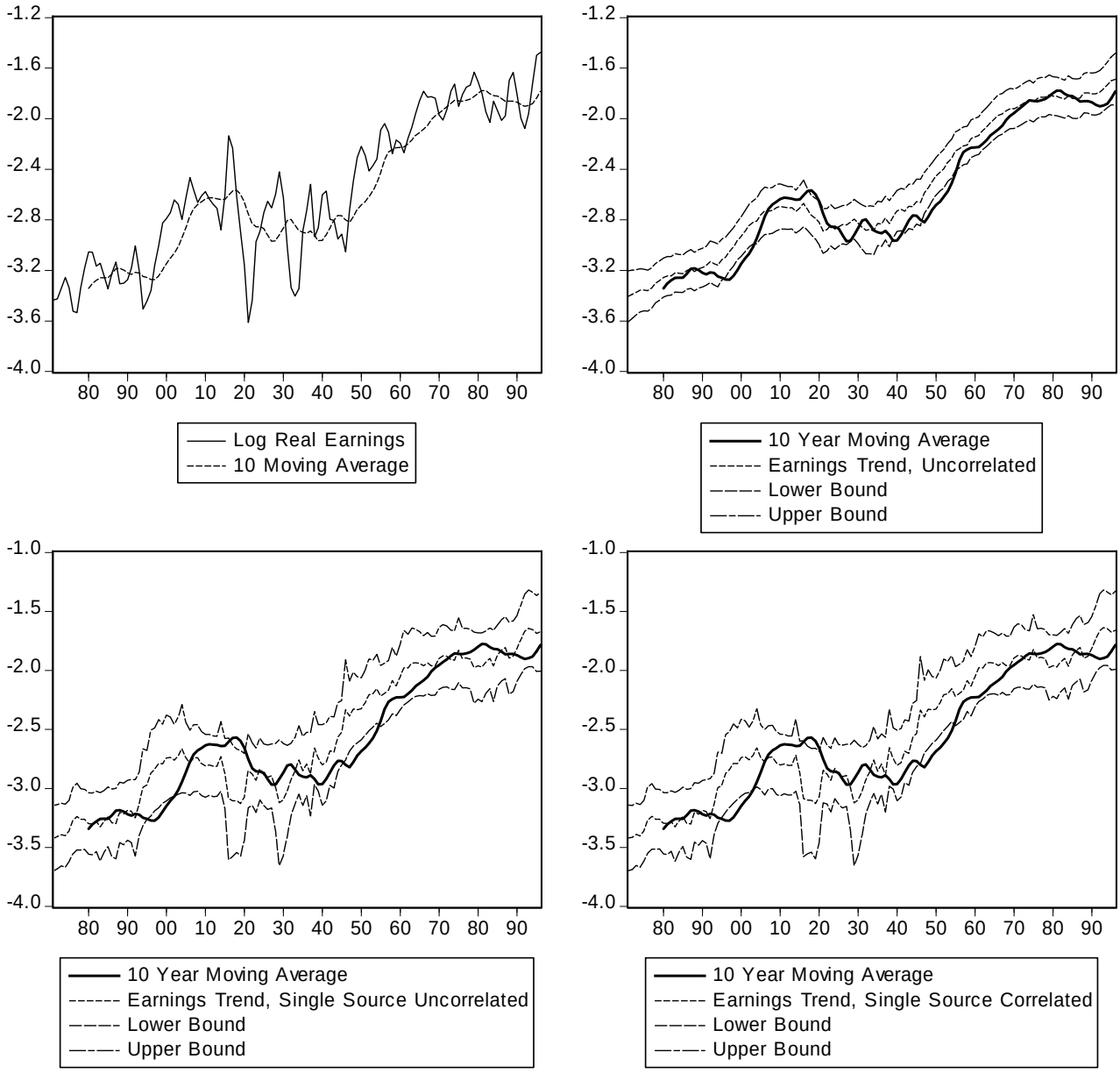
μ	σ_ε	σ_ν	R^2	SIC
0.014 (0.01)	0.065 (0.02)	0.141 (0.01)	0.287	-1.189

Panel D: The UC Model with Single Shock for Earnings and Correlated with Stock Returns

μ	σ_ε	σ_ν	$\rho_{\varepsilon\nu}$	R^2	SIC
0.014 (0.01)	0.067 (0.02)	0.135 (0.01)	-0.227 (0.09)	0.284	-1.193

Note: The numbers in the parentheses are standard errors. The R^2s are computed by regressing the 10 year future stock returns on a constant and the difference of log real stock price and estimated trend from the given model. The SIC s represent Schwarz Information Criterion and are computed for the bivariate UC models only.

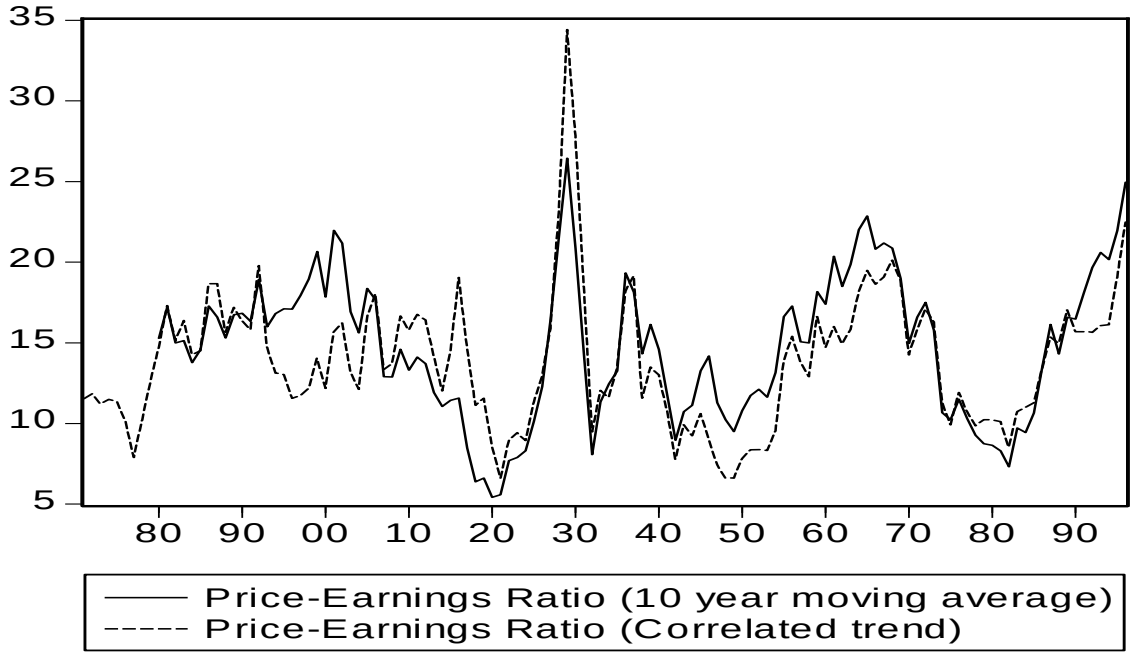
Figure 1: Estimates of Earnings Trends from Unobserved Components Models



Note: The upper-left panel shows the logarithm of real earnings data and its 10-year moving average. The upper-right panel shows the estimated log real earnings trend and its 90 percent confidence interval when the shocks are uncorrelated with each other. The lower-left panel shows the estimated log real earnings trend and its 90 percent confidence interval when the permanent shocks to real earnings are perfectly correlated to its transitory shocks but uncorrelated to future stock returns shock. The lower-right panel shows the estimated log real earnings trend and its 90 percent confidence interval when the permanent shocks to real earnings are perfectly correlated to its transitory shocks and correlated to future stock returns shock. The standard errors of the trends are based on 1000 Monte Carlo replications.

Not for Publication Appendix: Notes for the Referees

1. The univariate Beveridge-Nelson decomposition of log real earnings yielded an R^2 of about 9 percent in the stock returns regression, lower than the valuation ratio using the ten-year moving average.
2. The correlation between the trend and the cycle shocks to real earnings was also estimated following Morley, Nelson and Zivot (2003). The estimates showed a perfectly negative correlation as in the Beveridge-Nelson decomposition. This created a problem for getting the standard error of the estimated trend following Hamilton (1986), as some parameters were on the border of the parameter space. So we decided to use the model with a single source of shocks following Anderson et al. (2006) that does the Beveridge-Nelson decomposition under the assumption of perfect correlation.
3. Hamilton's (1986) procedure for computing the confidence interval for unobserved components accounts for both parametric uncertainty and filtering uncertainty.
4. If we limit our sample to the 1871-1987 period, implying the last observation for the ten-year future stock returns covers the period 1987 to 1996, the fit of the regression increases to 0.31, similar to that of Campbell and Shiller (1998). The fit of the regression based on the estimated trend from the correlated model also increases to 0.43. The fit of the regression based on the estimated trend from the uncorrelated model increases to 0.45.
5. The following graph compares the estimated price-earnings ratio from correlated trends model with the price-earnings ratio based on the ten-year moving average of earnings.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Price-Earnings Ratio (10 year moving average)
Price-Earnings Ratio (Correlated trend)